The Dark Side of Timed Opacity

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Context

Need for Security in Transactional Systems

- Web-services: e-banking, online transactions
- id documents: biometric passport, Medicare Card
- e-voting systems

Different Types of Security

- Integrity: illegal actions cannot be performed by an unauthorized user
  Bank account management cannot be managed by a third party
- Availability: some actions must be available
  Withdrawing money from your bank account
- Privacy: information should remain hidden from some users
  PIN code

introduced in [Mazaré (WITS'2004), Bryans et al. (FAST'2005)]

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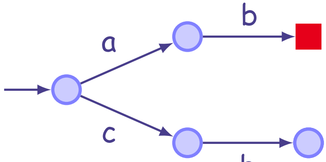
Formal Specification and Verification of Opacity

System S

Secret F



$\Sigma = \{a, b, c\}$

Secret = set of states ■
Events in $\Sigma_o \subseteq \Sigma$ are observable
Example: $\Sigma_o = \{b\}$

Opacity: an external observer should never know F-states

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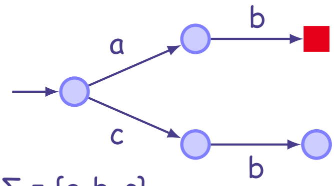

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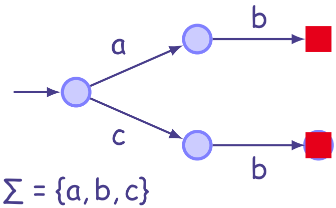

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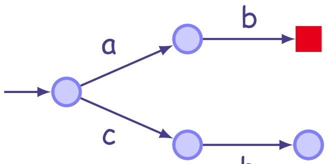

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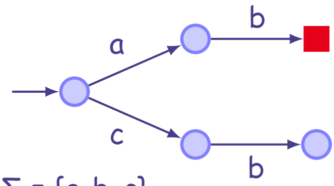

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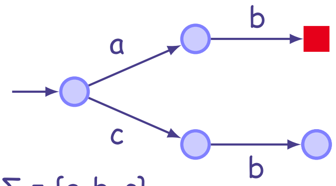

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To check opacity: use your favorite Formal Method:

- Model-checking
- Theorem proving
- Tools to support automatic analysis of systems

Results for Checking Opacity of Finite Systems

Inputs:

- S is finite automaton over alphabet Σ
- $\Sigma_o \subseteq \Sigma$, set of observable events
- a secret F, given by a subset of the set of states of S

Theorem ([Cassez et al. (ATVA'09)])

Checking wether F is opaque wrt (S, Σ_o) is PSPACE-complete.

What if an external observer can measure time ?

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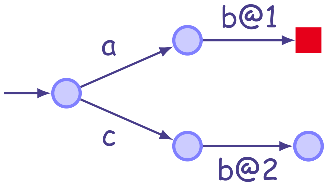

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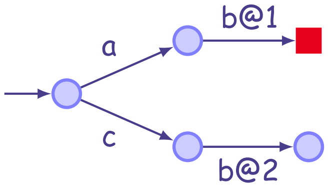

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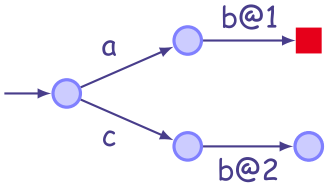

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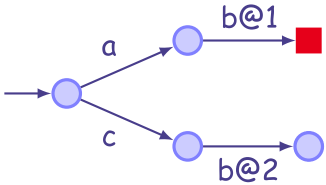

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Outline of the Talk

- Modelling Timed Systems
  - Timed Words and Languages
  - Timed Automata
  - Verification of Timed Automata
- Timed Opacity
  - Timed Opacity Problem
  - Timed Opacity is Undecidable for TA
- Conclusion

Timed Words and Languages

A finite timed word over Σ is a word in $(\Sigma \times \mathbb{R}_{\geq 0})^*$
(a, 1)(c, 2.34)(a, 2.986)(b, 3.146)(c, 4.16)

$TW^*(\Sigma)$ = set of timed words over Σ

Operations on timed words

- untiming: Unt(a, 1)(c, 2.34)(a, 2.986)(b, 3.146)(c, 4.16) = a.c.a.b.c
- Projection:
  $\pi_{\{a,b\}}((a, 1)(c, 2.34)(a, 2.986)(b, 3.146)(c, 4.16)) =$
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- Inverse Projection: $\pi^{-1}_{\Sigma}(w) = \{w' \in TW^*(\Sigma) \mid \pi_{\Sigma'}(w') = w\}$

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Operations on timed words extend to timed languages

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Timed Automata [Alur and Dill (TCS 94)]

- Timed Automaton = Finite Automaton + clock variables
  All clocks evolve at the same speed
- Clocks take their values in a dense-time domain
- Transitions are guarded by clocks constraints

- g: guard of the form $g ::= x \sim c \mid g \wedge g$
  where x is a clock and $c \in \mathbb{N}$, $\sim \in \{<, \leq, =, \geq, >\}$
- R : the set of clocks to be reset when firing the transition
- Inv(ℓ) is an invariant to ensure (some sort of) liveness

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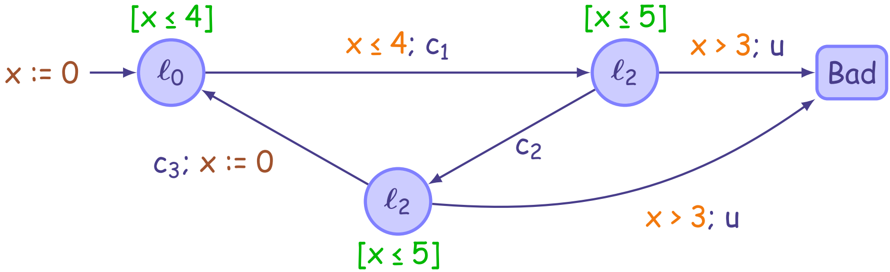

Runs = alternating sequence of discrete and time steps

ρ_1 : $(\ell_0, 0) \xrightarrow{1.55} (\ell_0, 1.55) \xrightarrow{c_1} (\ell_1, 1.55) \xrightarrow{1.67} (\ell_1, 3.22) \xrightarrow{u} (\mathrm{Bad}, 3.22)$

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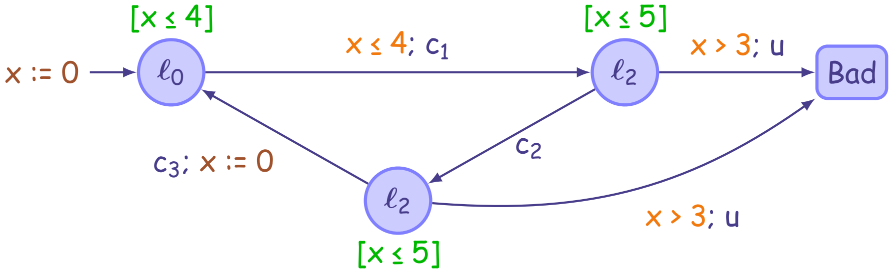

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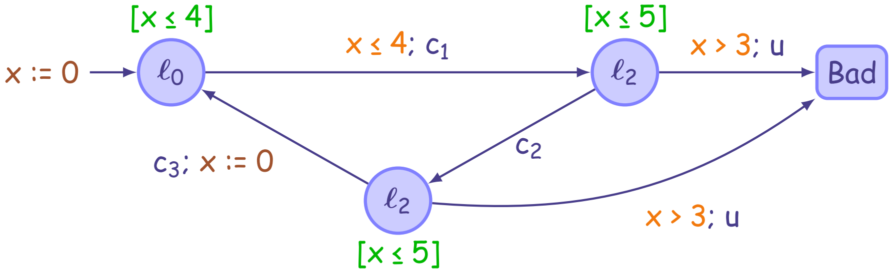

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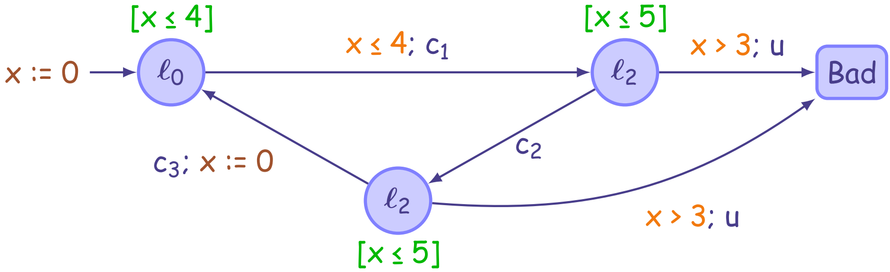

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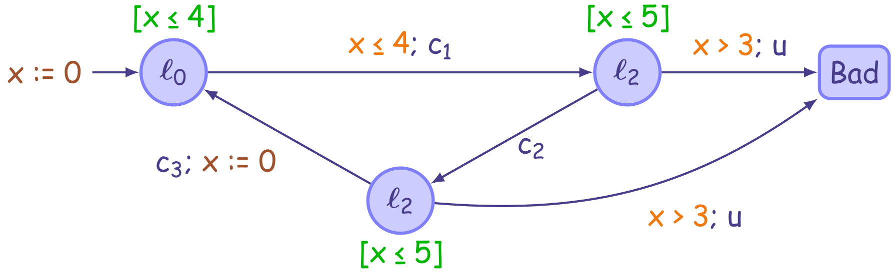

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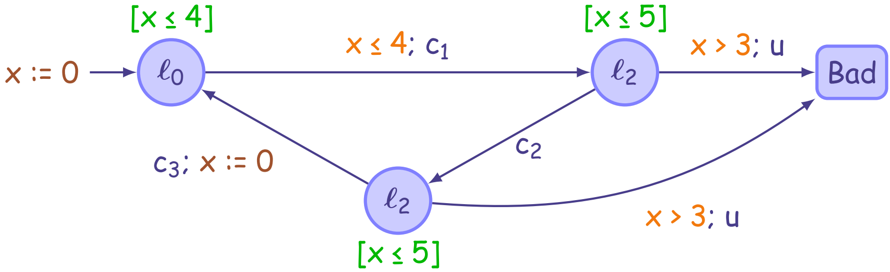

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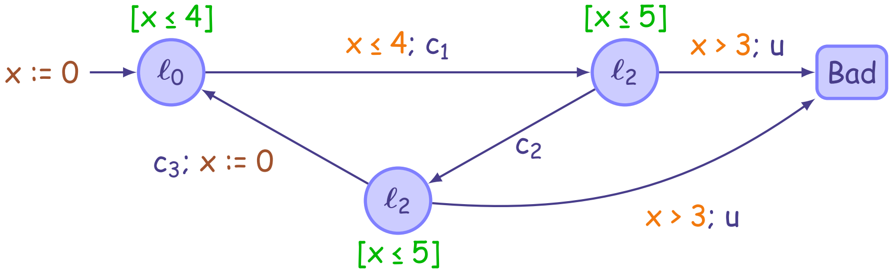

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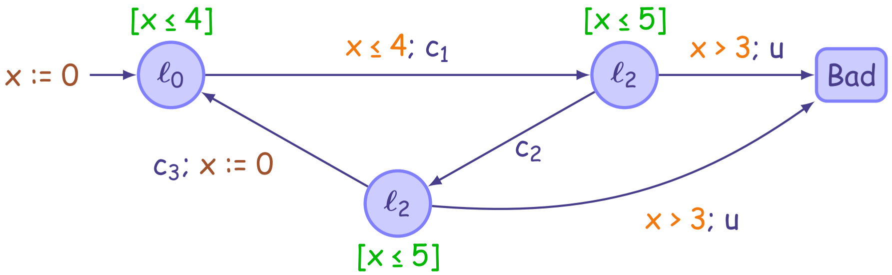

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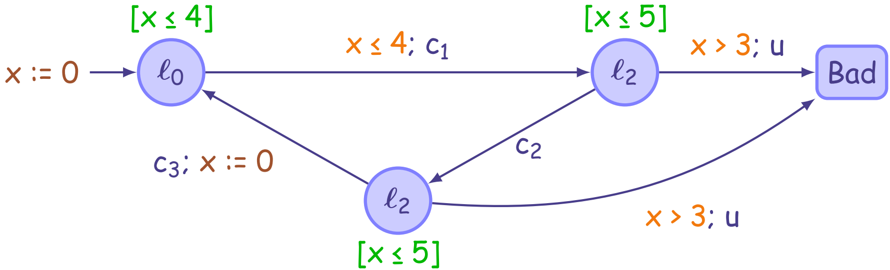

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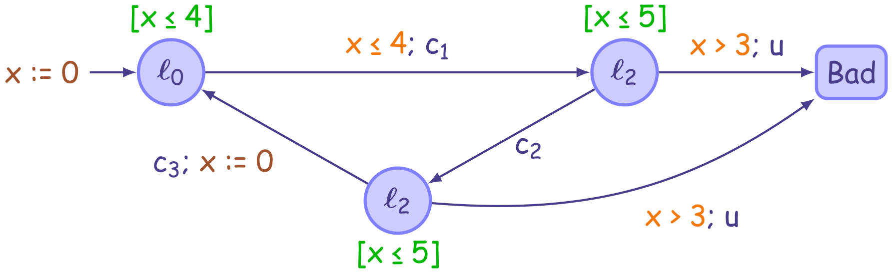

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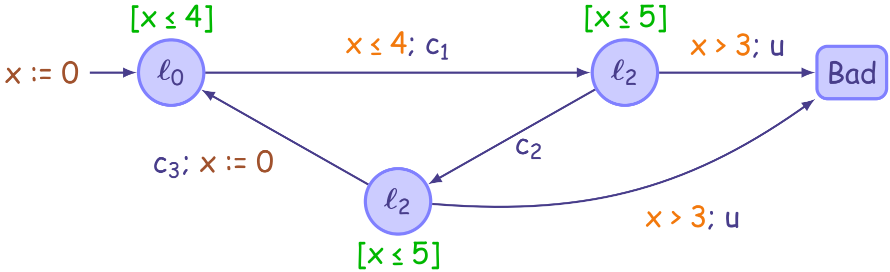

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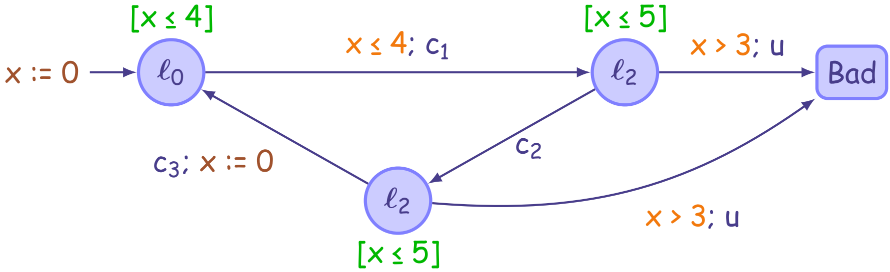

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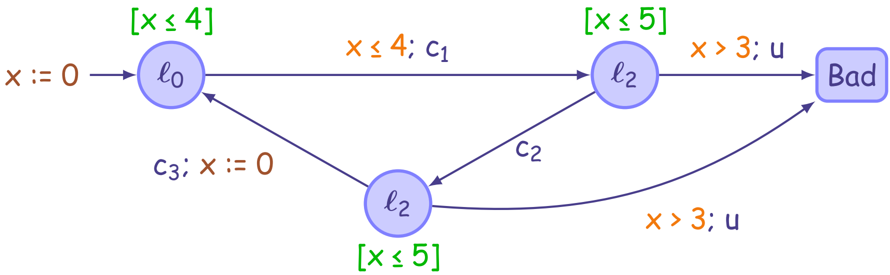

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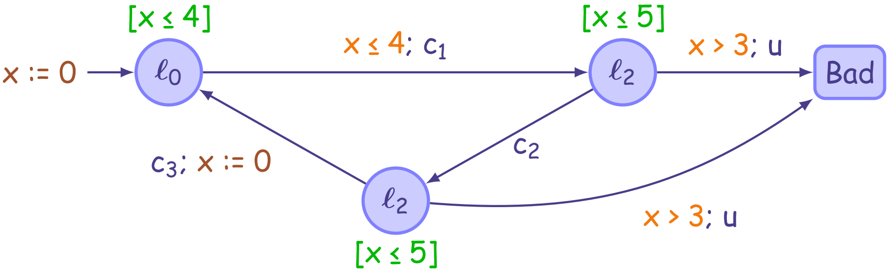

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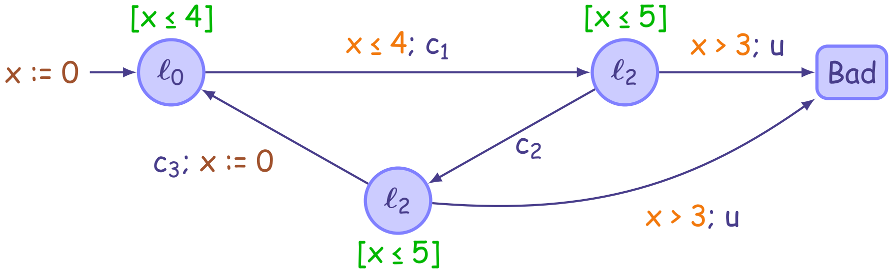

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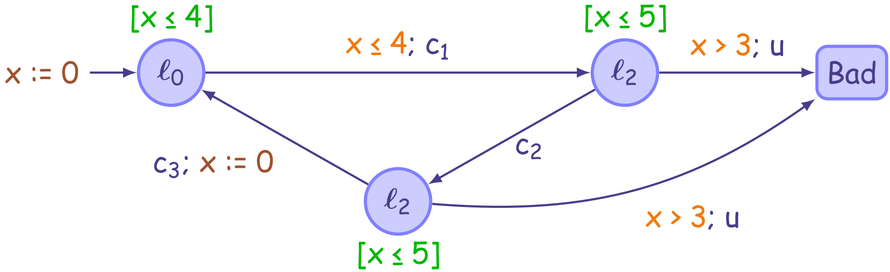

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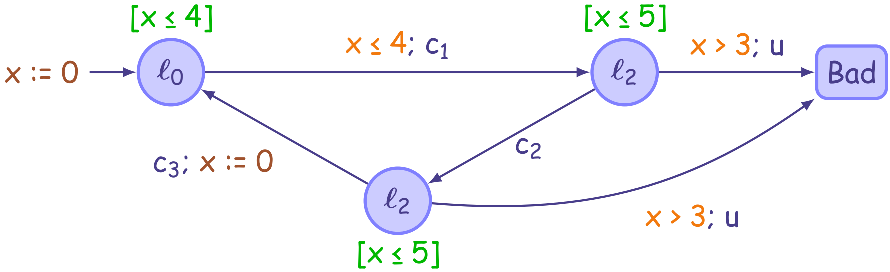

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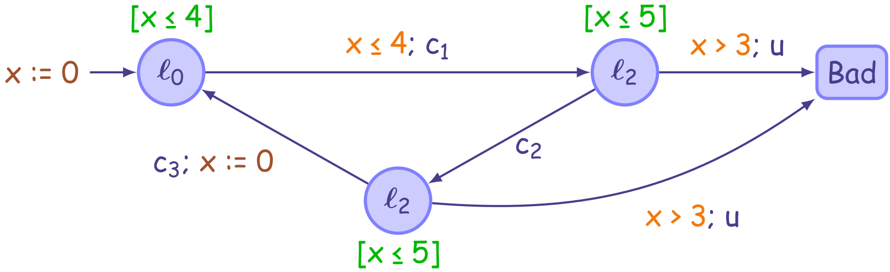

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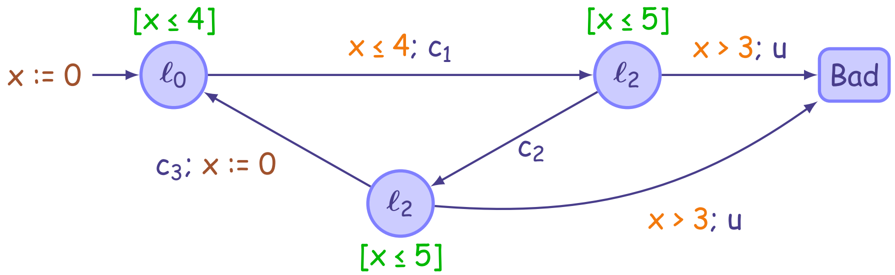

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Languages Generated by Timed Automata

A Timed Automaton A is a tuple $(L, \ell_0, X, \Sigma_\tau, E, F)$

$\Sigma_\tau = \Sigma \cup \{\tau\}$, τ = invisible/silent
F = subset of L, accepting locations

A run ϱ of A is a sequence of the form:

$$\begin{aligned}\varrho \quad = \quad & (\ell_0, v_0) \xrightarrow{\delta_0} (\ell_0, v_0 + \delta_0) \xrightarrow{a_0} (\ell_1, v_1) \cdots \\ & \cdots \xrightarrow{a_{n-1}} (\ell_n, v_n) \xrightarrow{\delta_n} (\ell_n, v_n + \delta_n)\end{aligned}$$

$tr(\varrho)$ is the trace of ϱ which is the timed word

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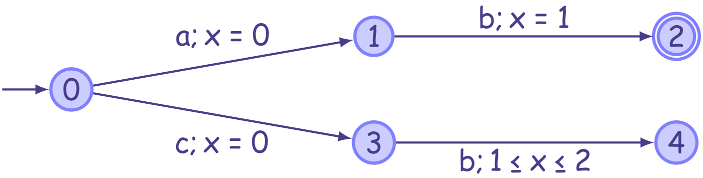

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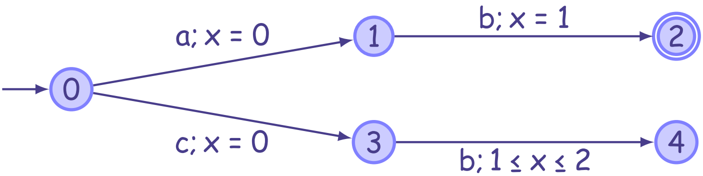

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- Timed Automata generate Timed Languages
  a timed word: $(a, 1.2)(b, 4.567)(a, 6)\cdots$
- Emptiness Problem: Is the language accepted by a TA empty ?
  reachability properties, Büchi-like properties
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Emptiness Problem for TA is PSPACE-Complete.
Build a finite time-bisimilar abstraction: region automaton

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Timed Opacity Problem

Given: a timed automaton $A = (L, \ell_0, X, \Sigma_T, E, F)$
F = set of secret locations
$\Sigma_o \subseteq \Sigma$, the set of observable actions

- $\pi(Tr(A))$ = set of projections on Σ_o of words generated by A
- $w \in \pi(Tr(A))$
  - $[w] = \pi^{-1}(w) \cap Tr(A)$
  - $last([w])$ set of locations A can be in after observing w

Definition (Opacity)

The secret F is opaque with respect to A and $\Sigma_o \subseteq \Sigma$ iff for each $w \in \pi(Tr(A))$, $last([w]) \not\subseteq F$.

Opacity Verification Problem for timed automata:

Check wether F is opaque w.r.t. (A, Σ_o).

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Results: Undecidability of Timed Opacity

Theorem

The opacity problem is undecidable for TA.

The proof is by reduction of the universality problem to the opacity problem.

Simpler Classes of Timed Automata

- Deterministic: no silent action and next state determined by (time,action)
- Event-Recording: deterministic, clocks are associated with actions [Alur et al. (CAV'94)]

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Conclusion & Further Results

Opacity + Dense-Time

- Checking Opacity is undecidable for TA
- Undecidability holds for simple timed systems like ERA
- Undecidability holds for time Petri Nets
  Timed automata and time Petri nets are equally expressive
  [Cassez and Roux (JSS 2006)]

Opacity + Discrete time

- Decidable but expensive

Opacity + Digital Clocks [Cassez and Tripakis (FI 2008)]

- A clock is a timed automaton (dense-time)
- Clock issues tick events
- External observer can only see $\Sigma_o \cup \{tick\}$
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Opacity + Dense-Time

- Checking Opacity is undecidable for TA
- Undecidability holds for simple timed systems like ERA
- Undecidability holds for time Petri Nets

  Timed automata and time Petri nets are equally expressive [Cassez and Roux (JSS 2006)]

Opacity + Discrete time

- Decidable but expensive

Opacity + Digital Clocks [Cassez and Tripakis (FI 2008)]

- A clock is a timed automaton (dense-time)
- Clock issues tick events
- External observer can only see $\Sigma_o \cup \{tick\}$
- Opacity with digital clocks is decidable in EXPTIME

Thanks !

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