Comparison of the Expressiveness of Timed Automata and Time Petri Nets

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FORMATS'05

Uppsala, Sweden

Outline of the talk

- Context & Motivation

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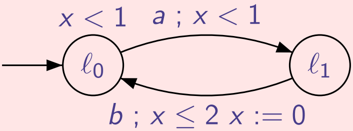
Timed Automata & Time Petri Nets

Timed Automata [Alur & Dill'94]

Timed Automata = Finite Automata + timing constraints given by clocks



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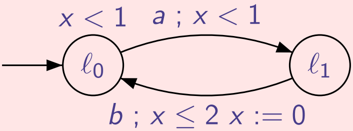

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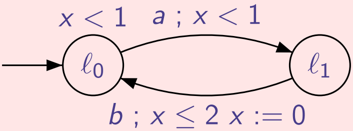

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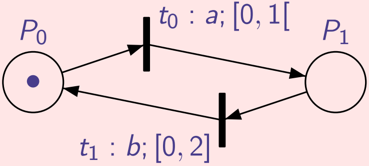

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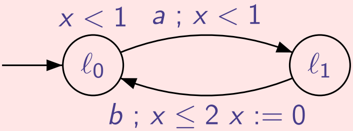

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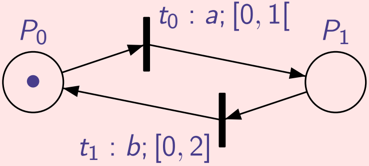

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Fundamental Problems for Timed Automata

We consider (Bounded) TPN introduced by [Merlin'74]

Problem	Timed Automata	B-Time Petri Nets
Reachability (RP) Emptyness (EP)		
Universality (UP) Language Inclusion		
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- Universality Problem
  checking a TPN against a spec. given by a TPN $\mathcal{L}(A) \subseteq \mathcal{L}(B)$
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  What about TPN ?
- TA or TPN as a specification language ?
  precise comparison of expressive power

Our Contribution

- Extended version of (bounded) TPNs: TPN_ε
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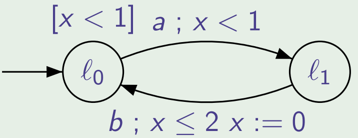
- Timed Bisimilarity: B-$TPN_\varepsilon(\leq, \geq)$ (original class defined by Merlin) and $TA_\varepsilon(\leq, \geq)$ are equivalent w.r.t. timed bisimilarity

Outline

Semantics of Timed Automata

Timed Automata

▸ Timed Automata



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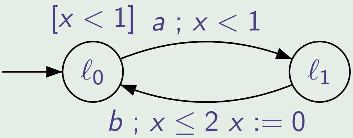

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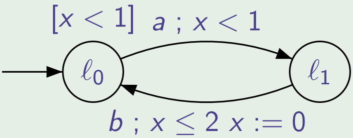

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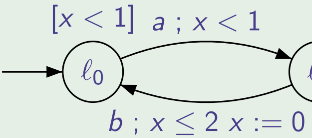

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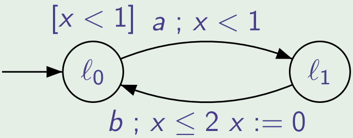

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  iff there is a transition (l, g, a, R, l') in $\mathcal{A}$ s.t.

$$\begin{cases} \textit{the guard is true in } (l, v) \\ v' \textit{ is } v \textit{ with the clocks in } R \textit{ equal to zero} \\ \textit{The invariant of } l' \textit{ holds for } v' \end{cases}$$

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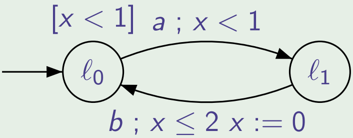

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  iff $\begin{cases} l = l' \text{ and } v' = v + t\text{: the clocks are updated} \\ \text{The invariant of } l \text{ holds for all } v + t', t' \leq t \end{cases}$

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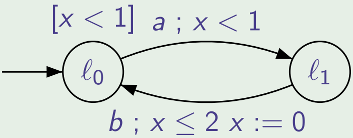

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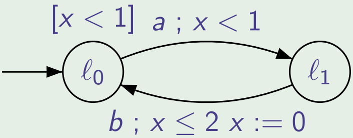

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- semantics of a TA A is a Timed Transition System S_A

Semantics of Time Petri Nets

Time Petri Net

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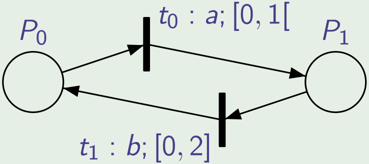

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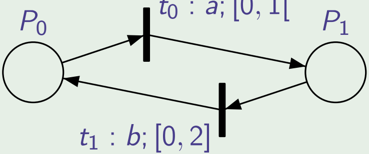

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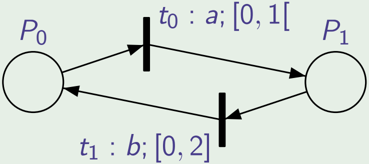

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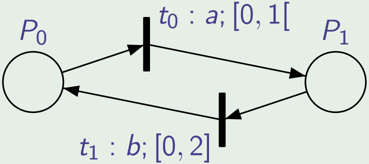

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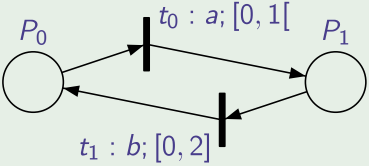

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- States: (M, ν) with M a marking and $\nu : Enabled(M) \mapsto \mathbb{R}_{\geq 0}$
- Discrete transition: $(M, \nu) \xrightarrow{a} (M', \nu')$ iff $\exists t : a, I$ in the s.t.

$$\begin{cases} t \text{ is enabled in } M \text{ and } M' = M - {}^\bullet t + t^\bullet \\ \nu(t) \text{ is in } I \text{ (the interval associated with } t) \\ \nu'(t') = 0 \text{ if } t' \text{ is enabled when firing } t, \ \nu'(t') = \nu(t') \text{ otherwise} \end{cases}$$

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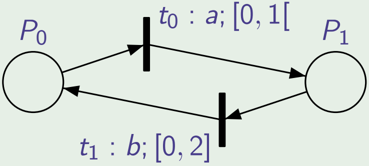

run 1: $(P_0, 0) \xrightarrow{0.78} (P_0, 0.78) \xrightarrow{a} (P_1, 0.78) \xrightarrow{1.5} (P_1, 1.5) \xrightarrow{b} (P_0, 0) \cdots$

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Definition (Semantics of Time Petri Nets)

- States: (M, ν) with M a marking and $\nu : Enabled(M) \mapsto \mathbb{R}_{\geq 0}$
- Discrete transition: $(M, \nu) \xrightarrow{a} (M', \nu')$
- Time transition: $(M, \nu) \xrightarrow{d} (M', \nu')$ iff
$$\begin{cases} M = M' \text{ and } \nu' = \nu + d \text{ (clocks of enabled transitions updated} \\ \text{For all enabled } t\text{, for all } d' \leq d,\ \nu(t) \in I(t) \end{cases}$$

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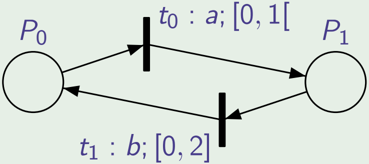

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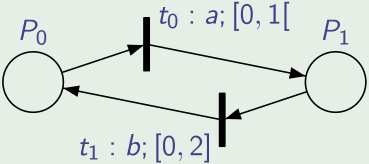

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- A TPN generates a set of runs = alternating discrete and time steps
- Semantics of a TPN N = Timed Transition System S_N

Outline

- Context & Motivation
- Timed Automata & Time Petri Nets
- **Expressiveness wrt Timed Bisimilarity**
- Expressiveness wrt Timed Language Acceptance
- Conclusion

Timed Bisimilarity

Definition (Weak Timed Bisimilarity)

Two TTS A and B are timed bisimilar if there is an equivalence relation $\equiv$ on the states of S_A and S_B s.t.: $s_0^A \equiv s_0^B$ and

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1. for each state s of S_A there is a state q of S_B s.t. $s \equiv q$ and if $s \xrightarrow{\sigma} s'$ then $q \xrightarrow{\sigma} q'$ and $s' \equiv q'$ and s'
2. for each state q of S_B ...

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Weakly Timed Bisimilar: allows ε-moves

- discrete step: $s \xRightarrow{a} s'$ if $s \xrightarrow{\varepsilon^*}\xrightarrow{a}\xrightarrow{\varepsilon^*} s'$
- time step: $s \xRightarrow{\delta} s'$ if $s \xrightarrow{\varepsilon^*}\xrightarrow{\delta_1}\xrightarrow{\varepsilon^+} \cdots \xrightarrow{\varepsilon^+}\xrightarrow{\delta_n}\xrightarrow{\varepsilon^*} s'$ and $\sum \delta_i = \delta$

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Theorem ([C. & R., AVoCS'04])

Each bounded TPN (TPN_ε) is timed bisimilar to a TA (TA_ε).

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Theorem ([C. & R., AVoCS'04])

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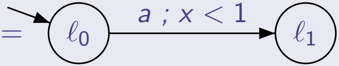
Converse: Each TA is timed bisimilar to a TPN ?

Theorem (TA are strictly more expressive than TPNs)

Let $\mathcal{A}_0 =$



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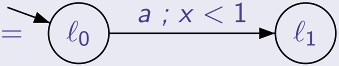

There is **no TPN** *weakly timed bisimilar to* $\mathcal{A}_0$.

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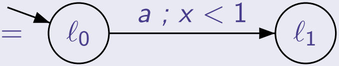

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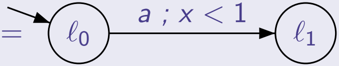

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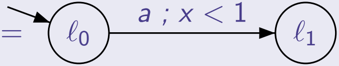

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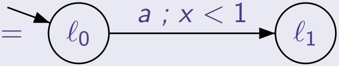

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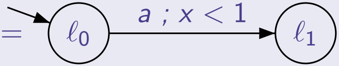

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$$(\ell_0, x = 0) \xrightarrow{\delta = 1} (\ell_0, x = 1)$$

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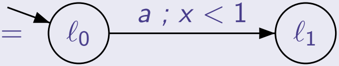

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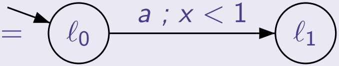

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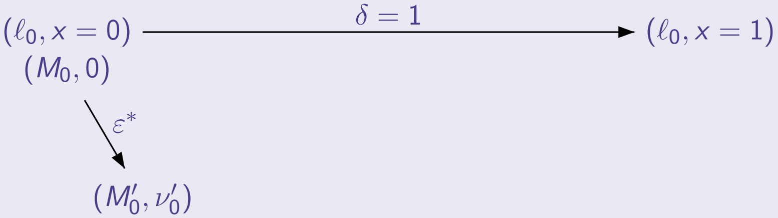

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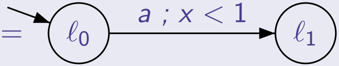

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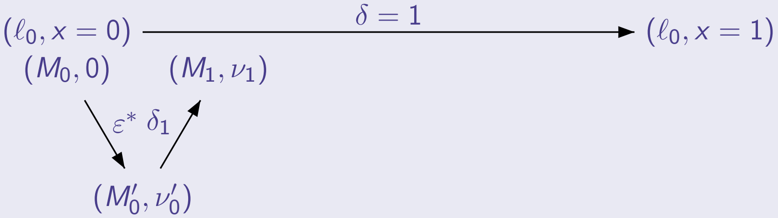

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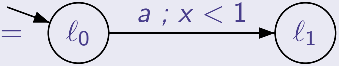

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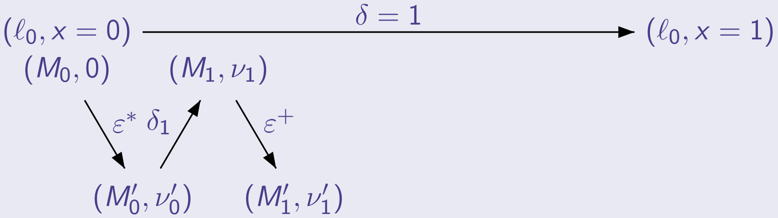

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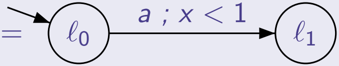

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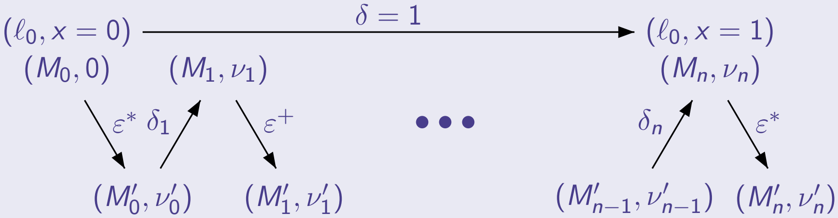

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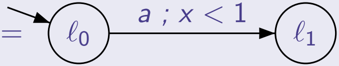

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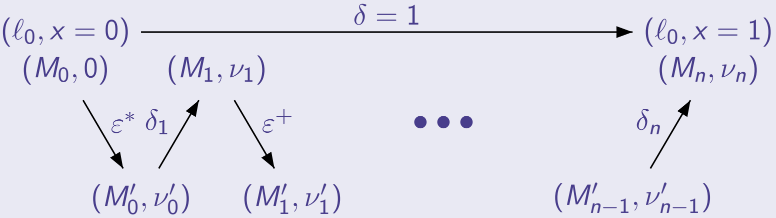

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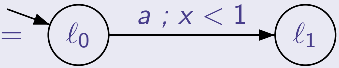

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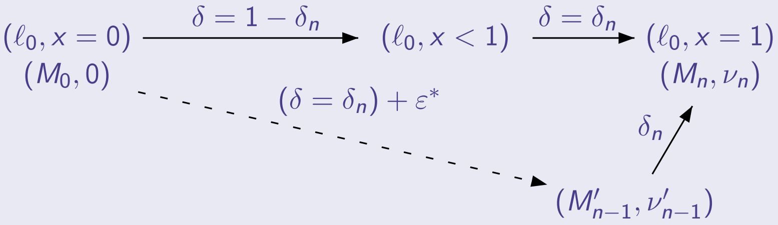

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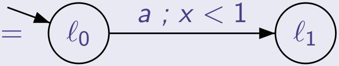

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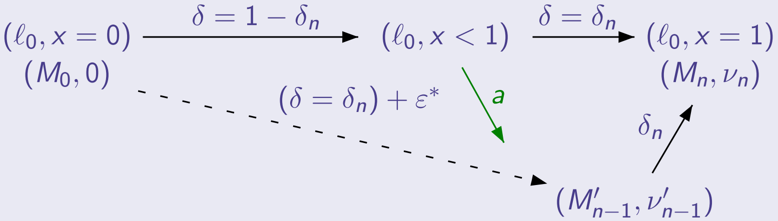

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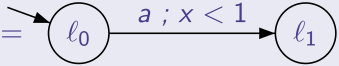

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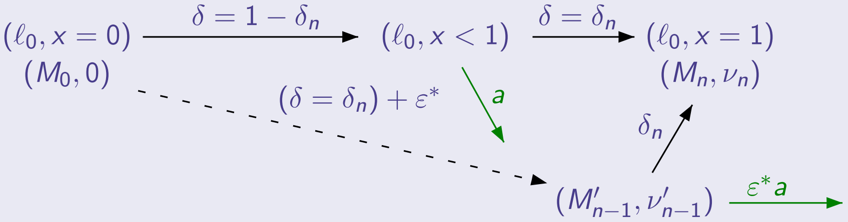

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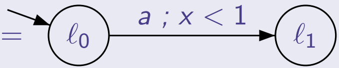

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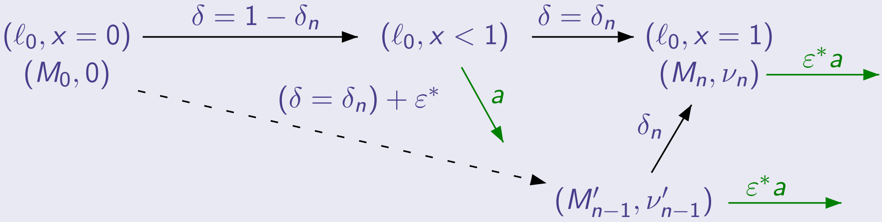

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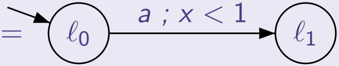

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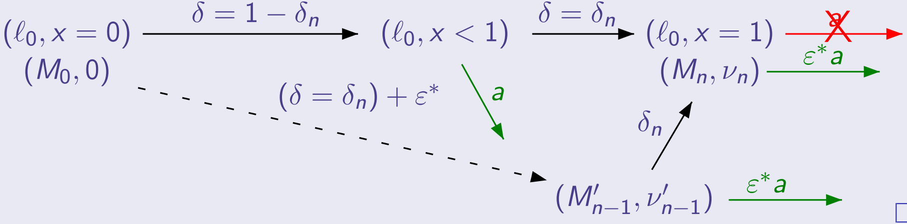

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Language Equivalence

Definition (Timed Word & Timed Language)

A timed word over Σ is a sequence $w = (a_0, \delta_0)(a_1, \delta_1) \cdots (a_n, \delta_n) \cdots$ with $a_i \in \Sigma$; $\delta_i \in \mathbb{R}_{\geq 0}$.

A timed language is a set of timed words.

A TTS A with final and repeated states accepts a timed language $\mathcal{L}(A)$.

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A TTS A with final and repeated states accepts a timed language $\mathcal{L}(A)$.

Definition (Language Equivalence)

Two TTS A and B are equivalent w.r.t. Timed Language Acceptance if

$$\mathcal{L}(A) = \mathcal{L}(B).$$

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A TTS A with final and repeated states accepts a timed language $\mathcal{L}(A)$.

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Two TTS A and B are equivalent w.r.t. Timed Language Acceptance if

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Theorem ([C. & R., AVoCS'04])

A timed language accepted by a bounded TPN is accepted by a TA.

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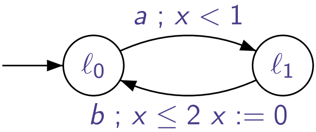
Each language accepted by a TA is accepted by a TPN ?

Encoding a TA into a Bounded TPN



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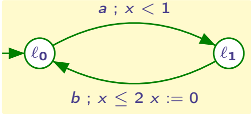

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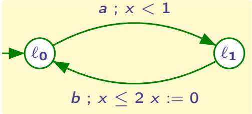

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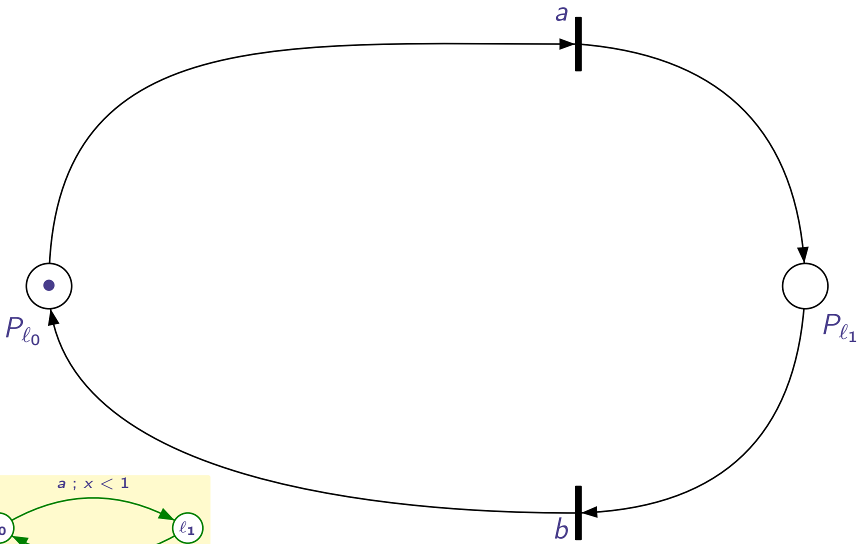

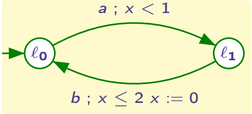

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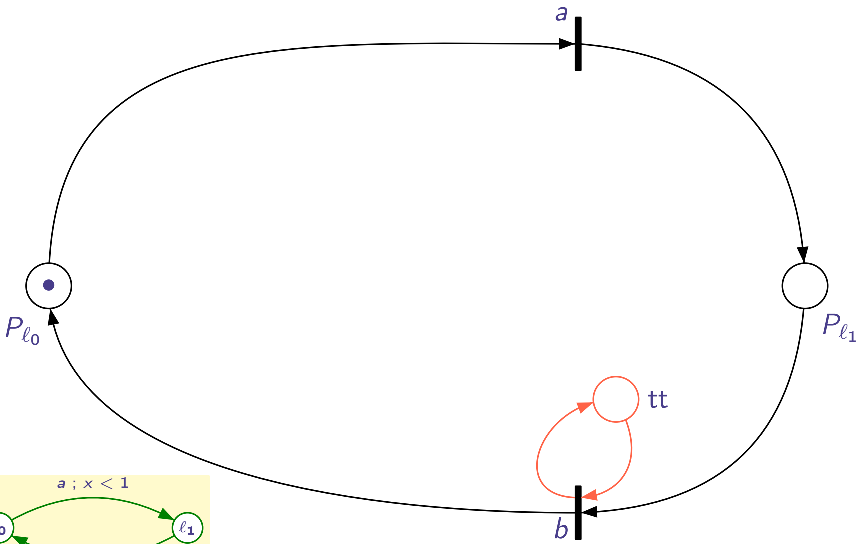

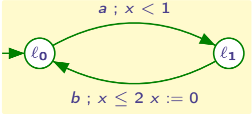

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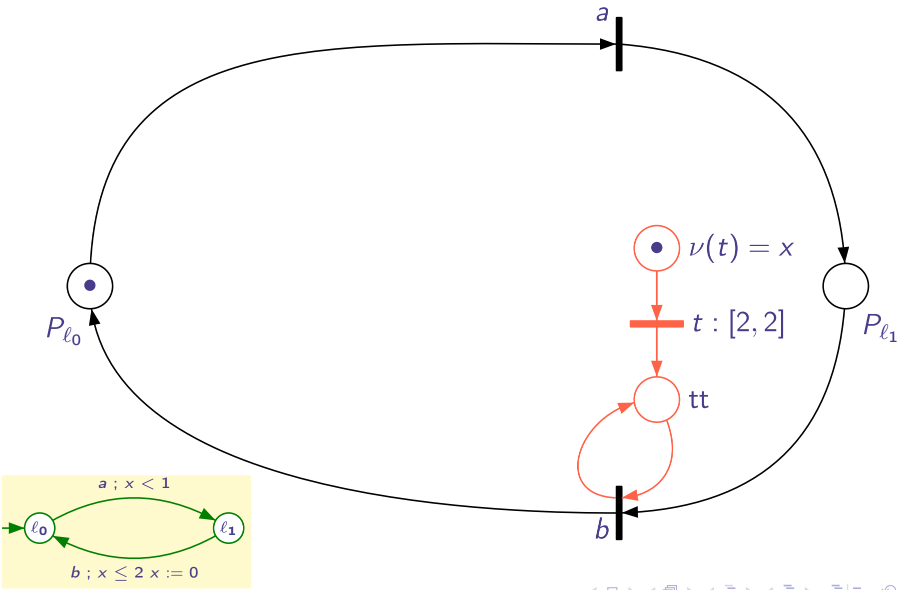

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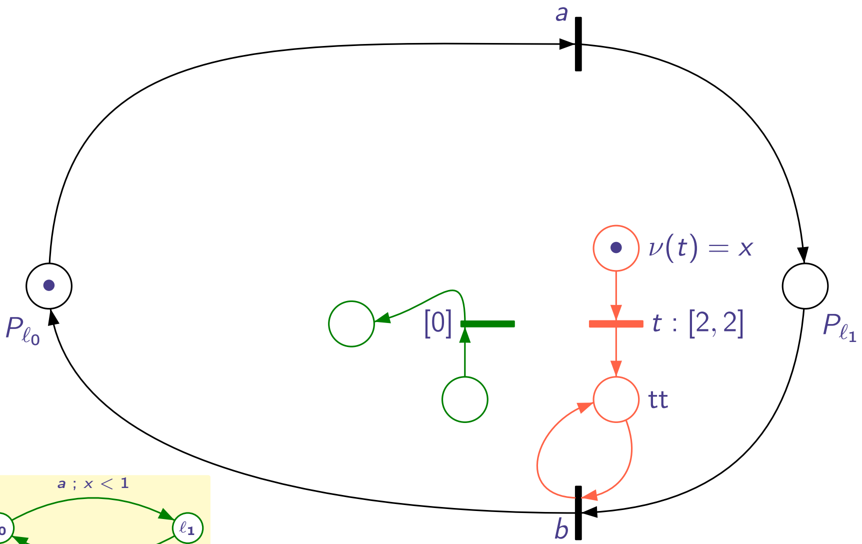

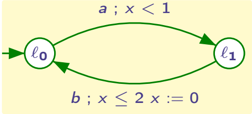

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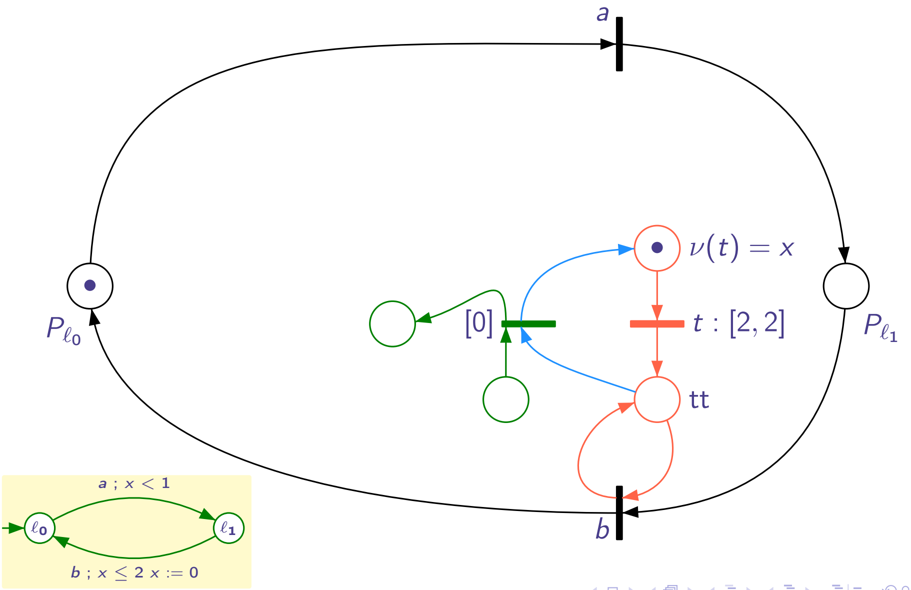

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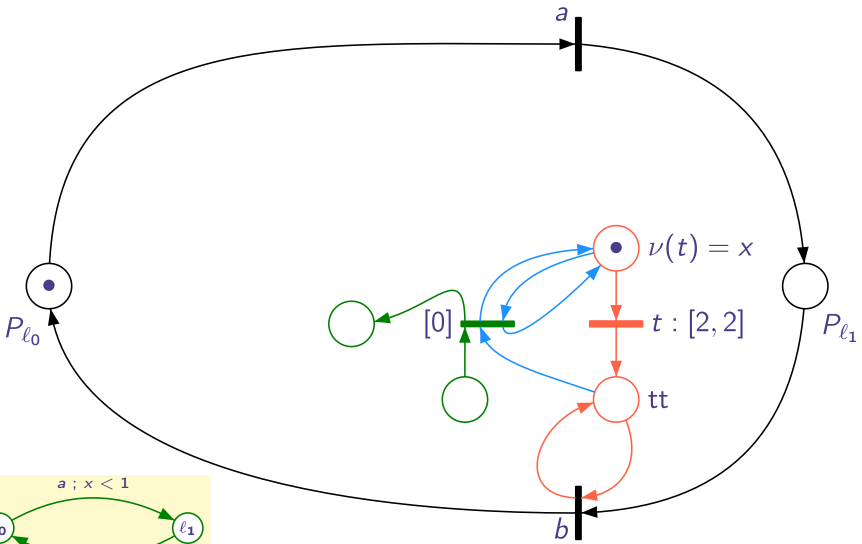

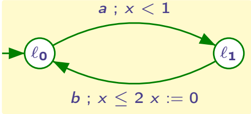

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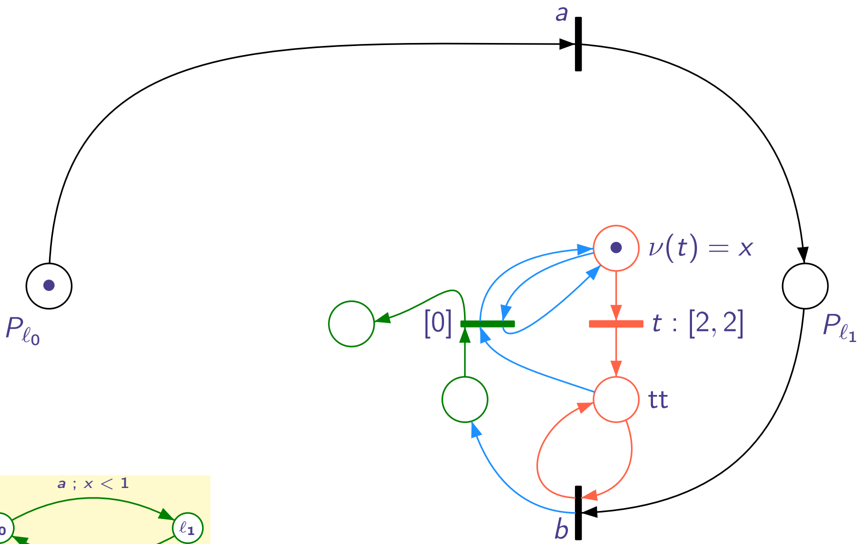

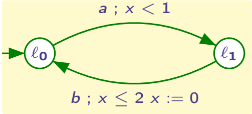

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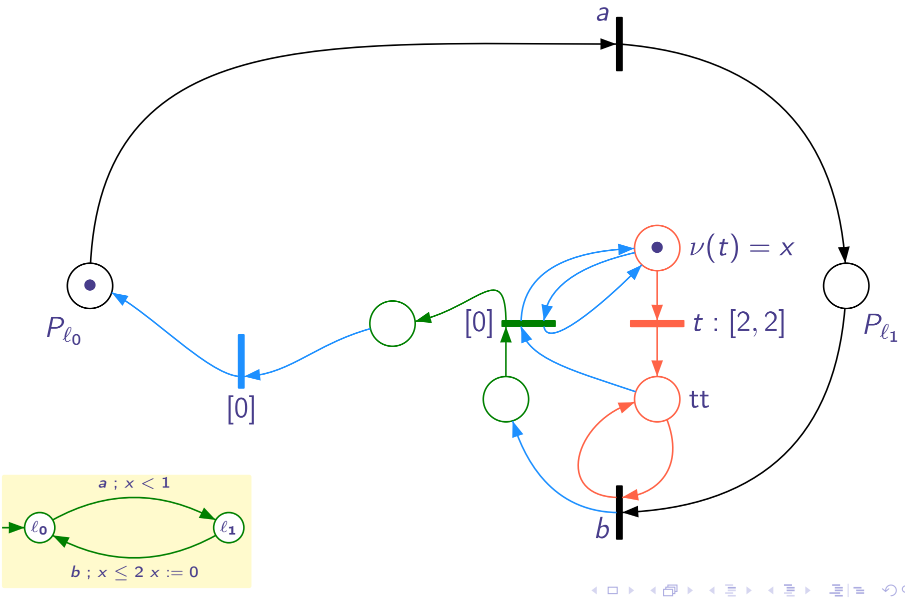

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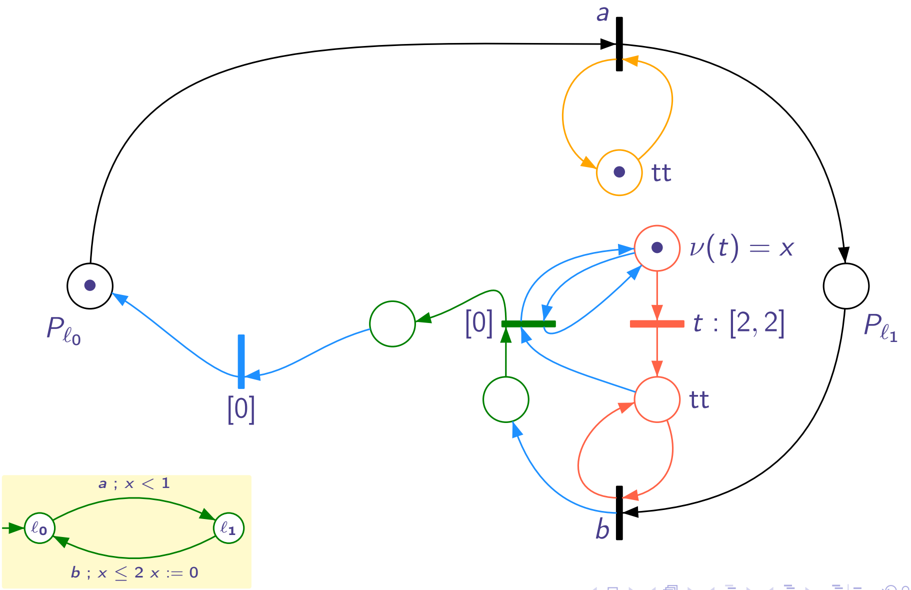

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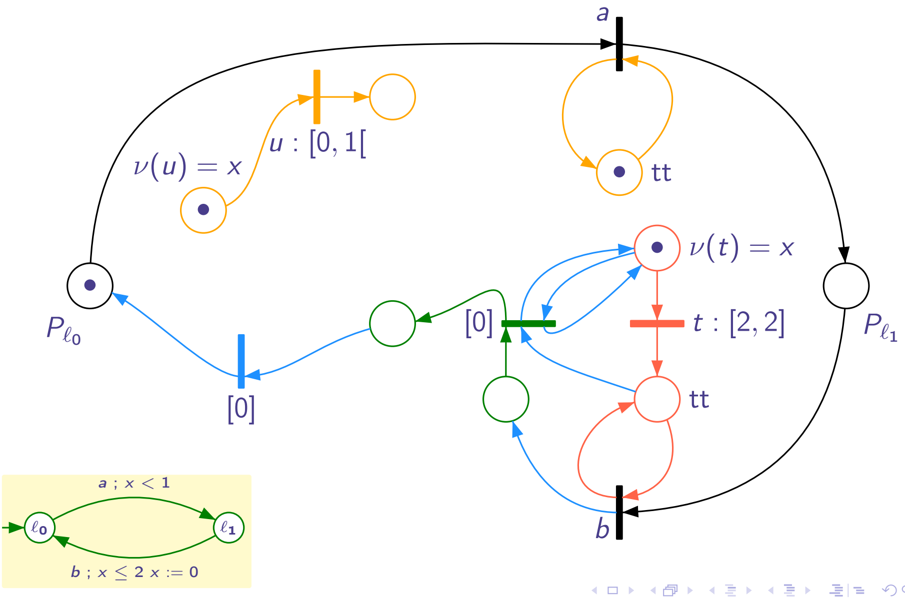

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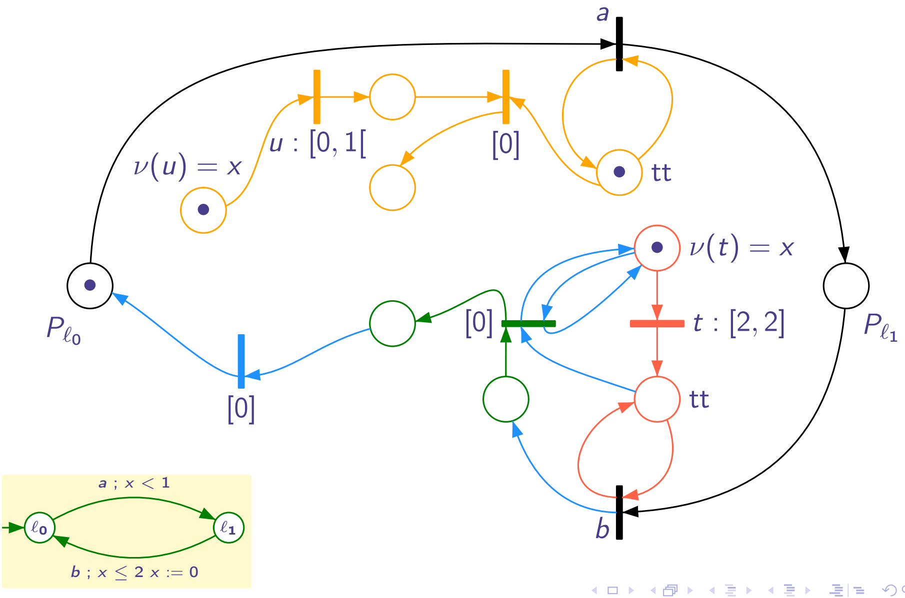

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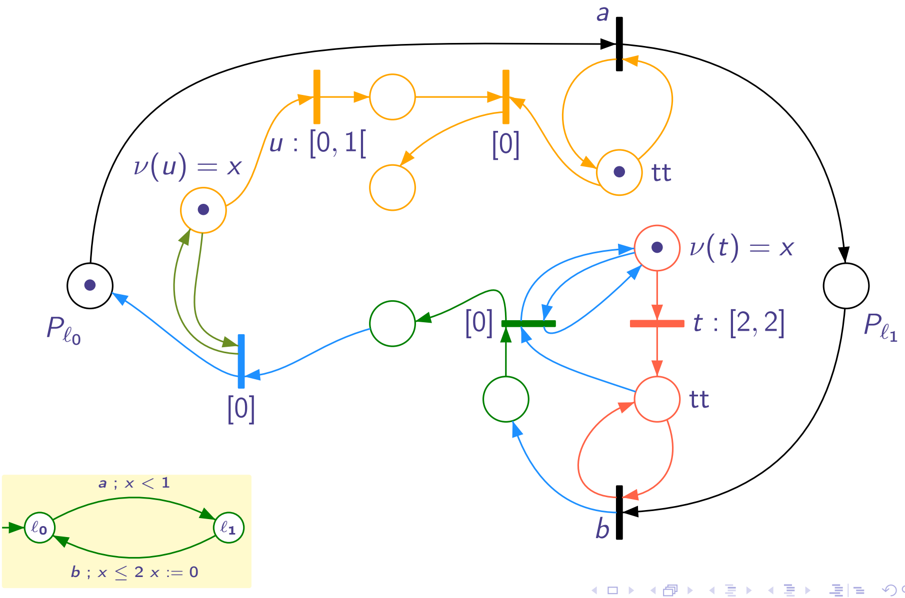

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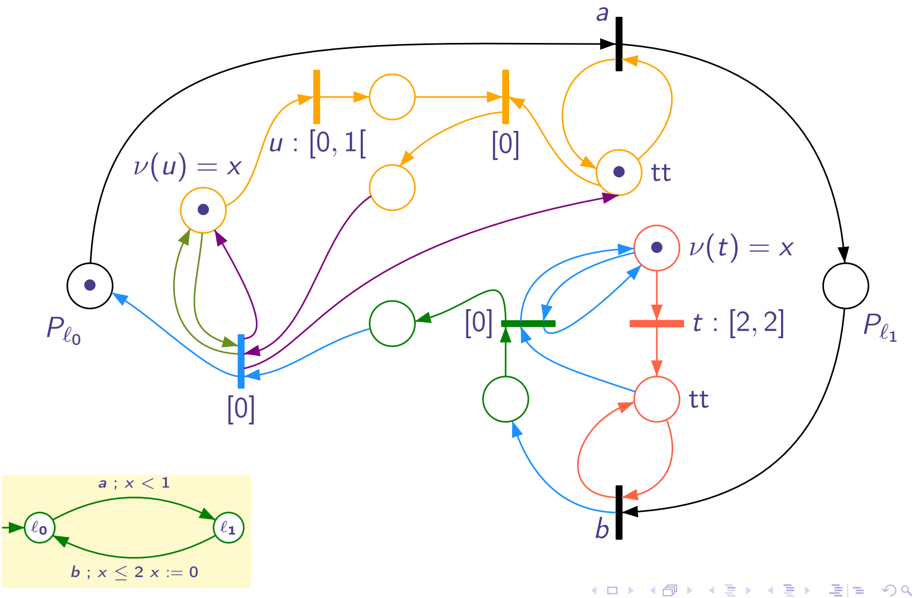

TA and TPN are equally Expressive

Theorem

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Sketch.

$\mathcal{A}$ a TA; $\mathcal{N}$ the TPN as described previously.
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□

Back to Timed Bisimilarity

Definition (The class $TA_\varepsilon(\leq, \geq)$)

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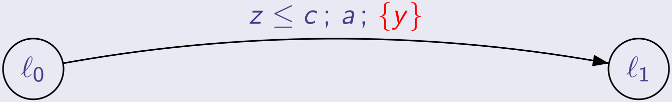

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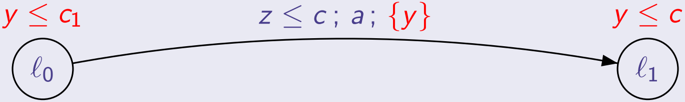

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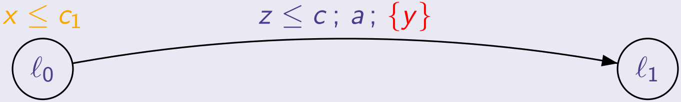

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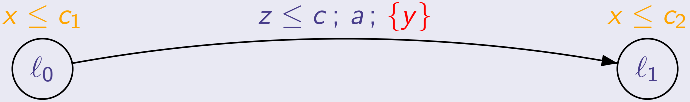

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- From B-$TPN_\varepsilon(\leq, \geq)$ to $TA_\varepsilon(\leq, \geq)$: [C. & R., AVoCS'04]

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  Extension of the previous construction for Timed Language

□

New Results for TPNs

Theorem (One is enough)

One-safe B-TPN$_\varepsilon$ and B-TPN$_\varepsilon$ are equally expressive w.r.t. timed language acceptance.

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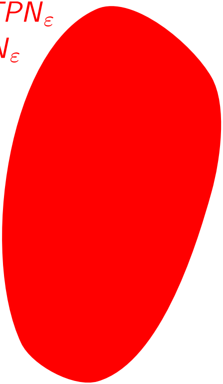
Current Picture

Timed Language Acceptance

$TA_\varepsilon = \text{B-}TPN_\varepsilon$
$= \text{1-B-}TPN_\varepsilon$



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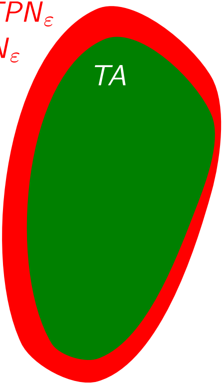

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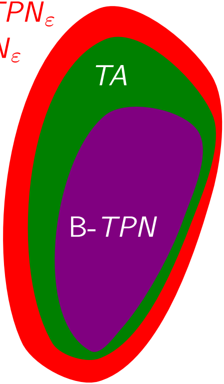

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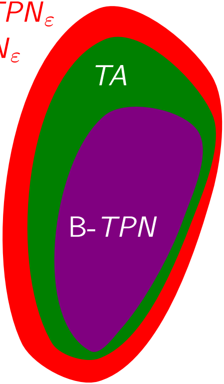

Q:B-$TPN = TA$?

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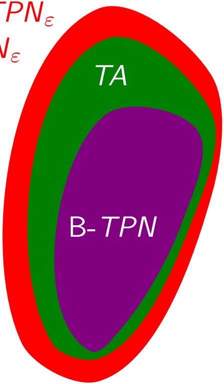

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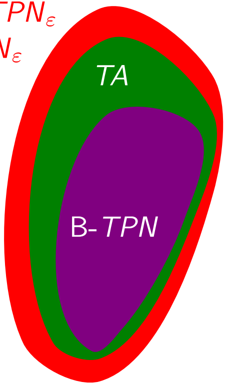

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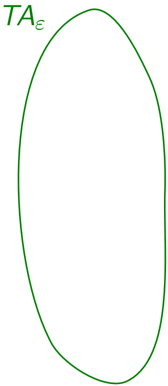

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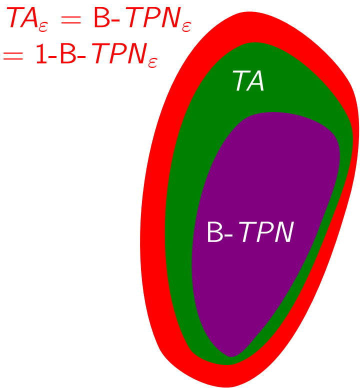

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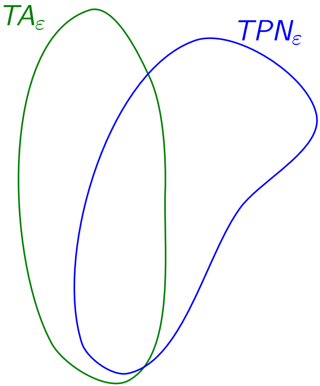

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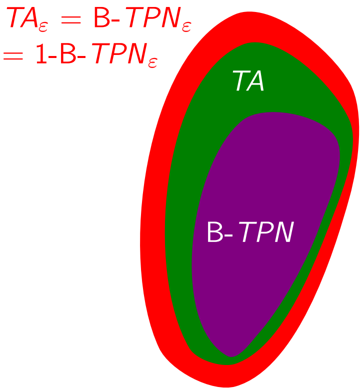

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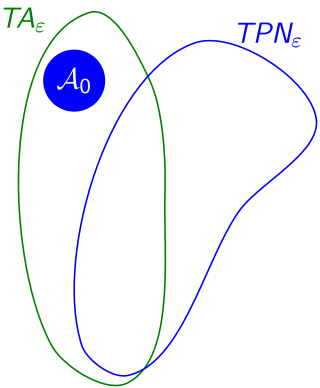

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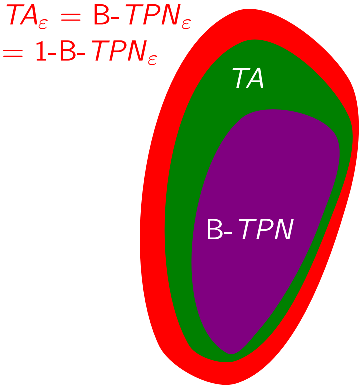

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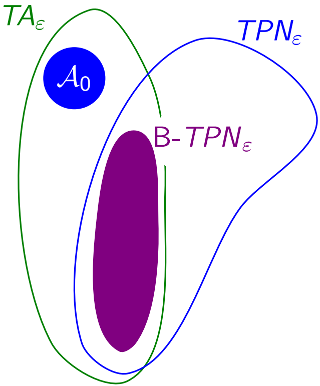

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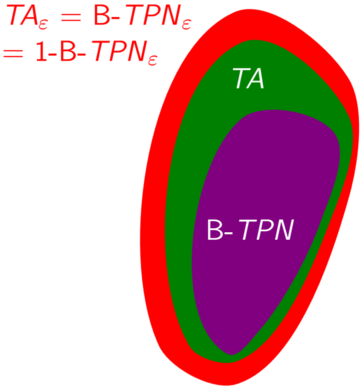

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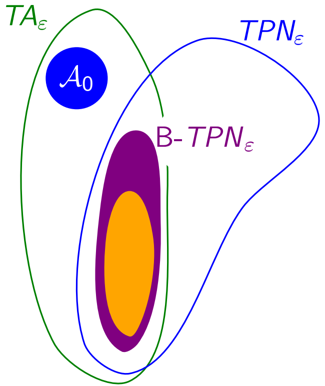

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Outline

- Context & Motivation
- Timed Automata & Time Petri Nets
- Expressiveness wrt Timed Bisimilarity
- Expressiveness wrt Timed Language Acceptance
- Conclusion

Conclusion & Recent Work

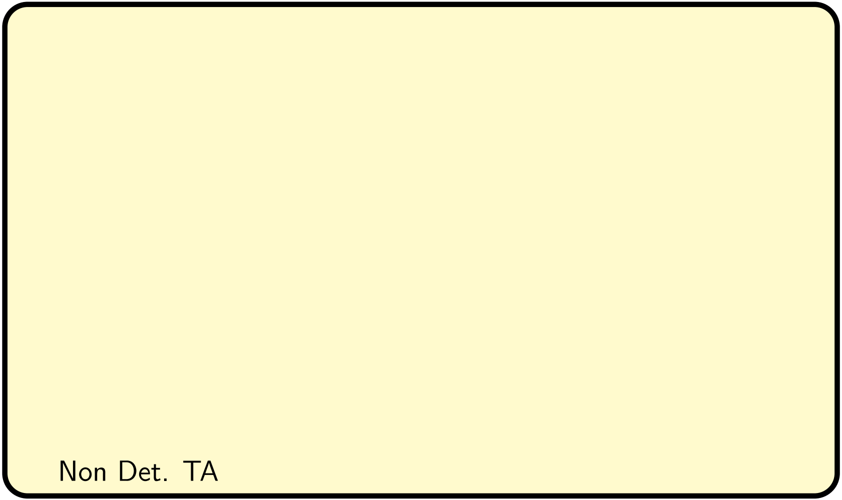
Results:

- Expressive Power of TA vs. TPNs
  Timed Language Acceptance and Timed Bisimilarity
- Undecidability of the Universal Problem
- Equivalence between one-safe TPN and TPN

Recent Results:

- more than one semantics for TPN
  [Bérard et al., ATVA'05]
- semantic definition of the class of TA that are timed bisimilar to TPN
  [Bérard et al., FSTTCS'05]

TPN and Event-Clock Automata [Alur et al.'99]



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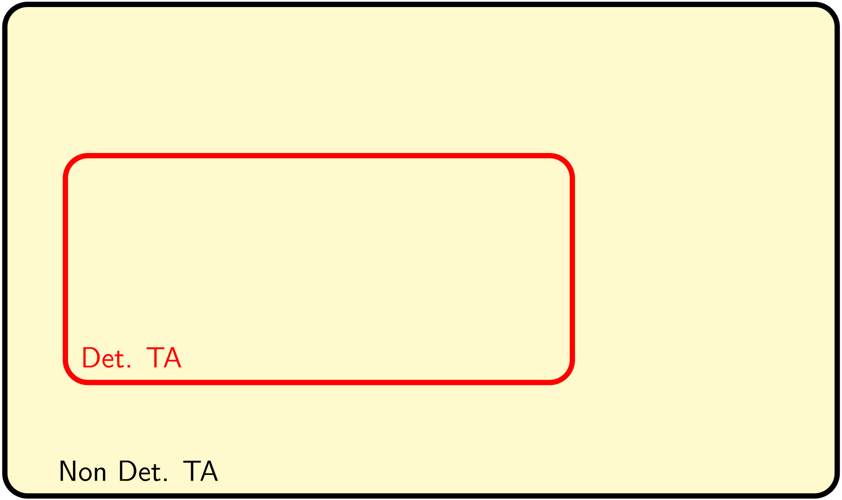

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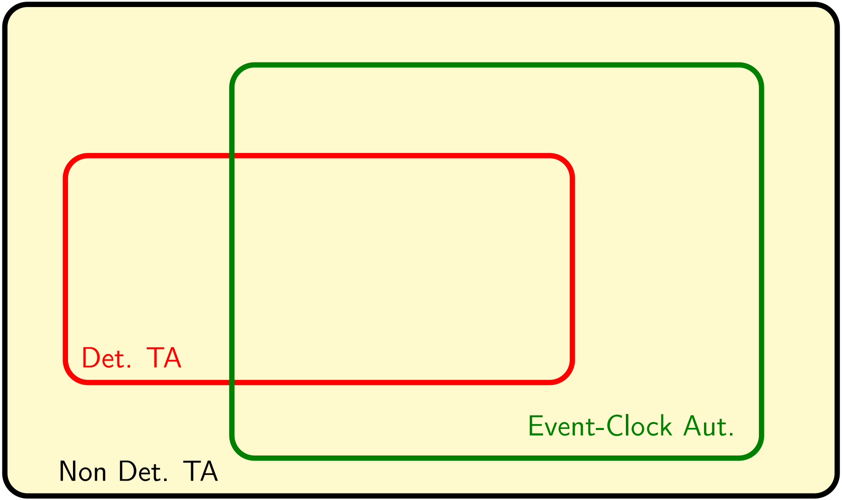

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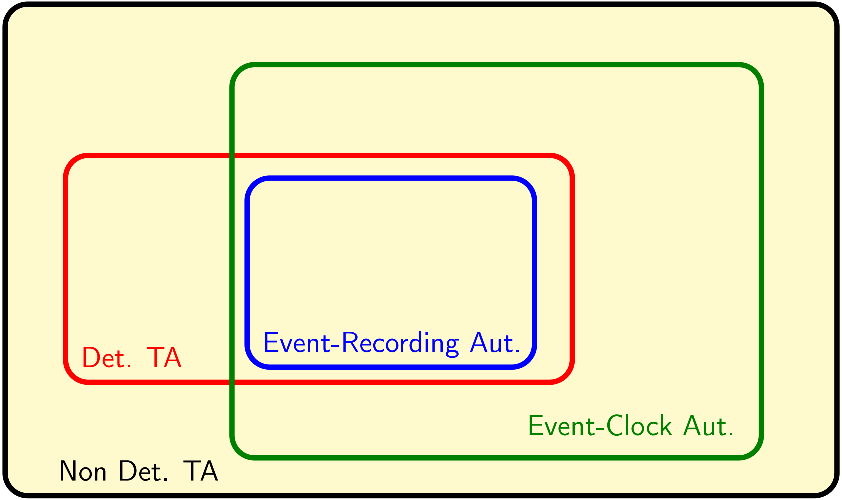

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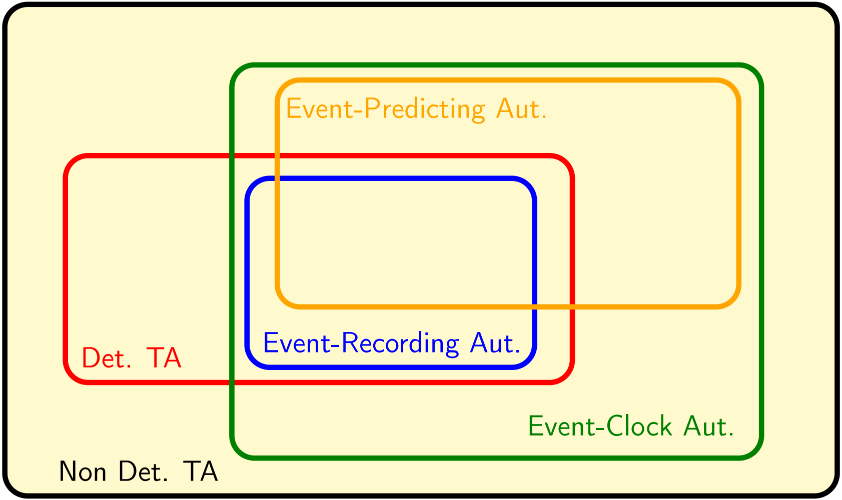

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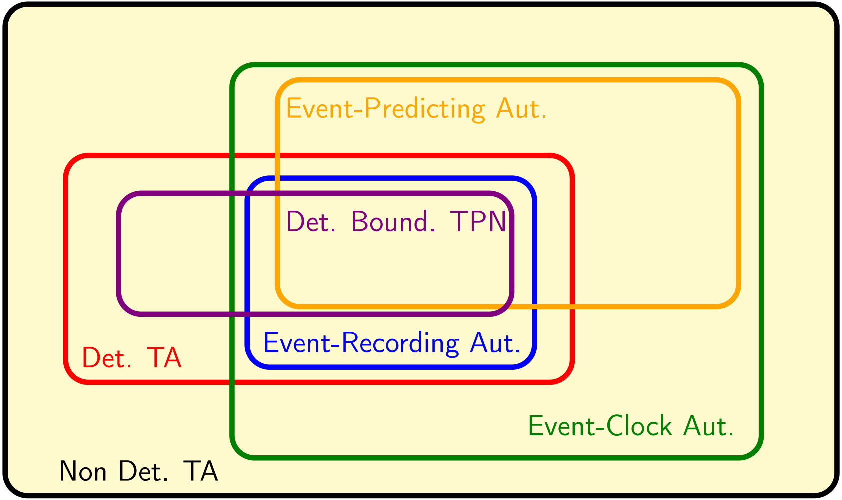

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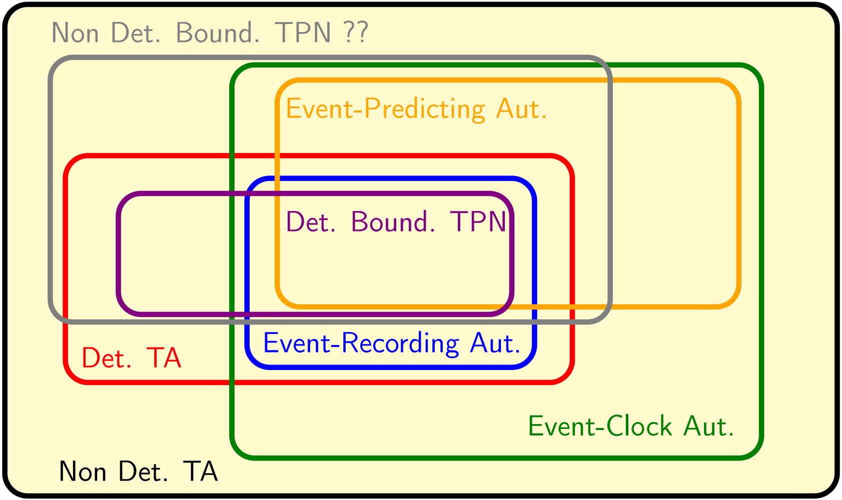

TPN and Event-Clock Automata [Alur et al.'99]

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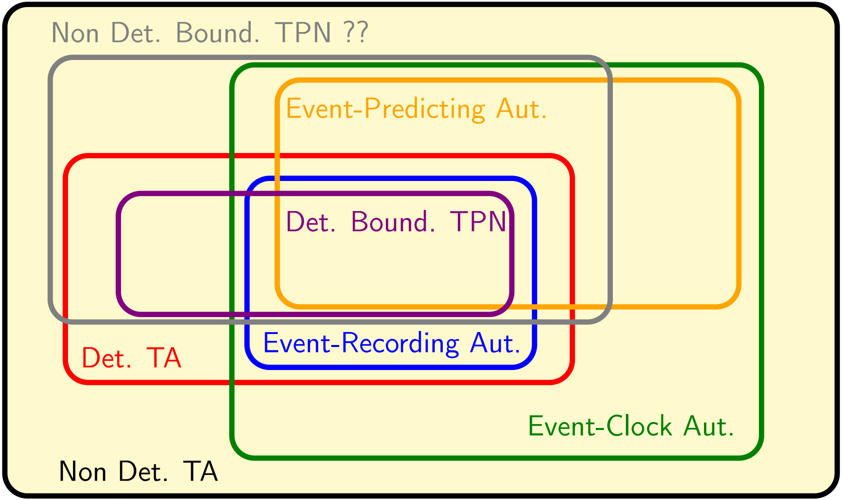

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Timed Automata [Alur & Dill'94]

Back

A Timed Automaton $\mathcal{A}$ is a tuple $(L, \ell_0, \mathrm{Act}, X, \mathrm{inv}, \longrightarrow)$ where:

- L is a finite set of locations
- ℓ_0 is the initial location
- X is a finite set of clocks
- Act is a finite set of actions
- $\longrightarrow$ is a set of transitions of the form $\ell \xrightarrow{g, a, R} \ell'$ with:
  - $\ell, \ell' \in L$,
  - $a \in \mathrm{Act}$
  - a guard g which is a clock constraint over X
  - a reset set R which is the set of clocks to be reset to 0

Clock constraints are boolean combinations of $x \sim k$ with $x \in C$ and $k \in \mathbb{Z}$ and $\sim \in \{\leq, <\}$.

Semantics of Timed Automata

Back

Let $\mathcal{A} = (L, \ell_0, \mathrm{Act}, X, \mathrm{inv}, \longrightarrow)$ be a Timed Automaton.

A state (ℓ, v) of $\mathcal{A}$ is in $L \times \mathbb{R}_{\geq 0}^X$

The semantics of $\mathcal{A}$ is a Timed Transition System $S_{\mathcal{A}} = (Q, q_0, \mathrm{Act} \cup \mathbb{R}_{\geq 0}, \longrightarrow)$ with:

- $Q = L \times \mathbb{R}_{\geq 0}^X$
- $q_0 = (\ell_0, \overline{0})$
- $\longrightarrow$ consists in:

discrete transition: $(\ell, v) \xrightarrow{a} (\ell', v') \iff \begin{cases} \exists\, \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A} \\ v \models g \\ v' = v[r \leftarrow 0] \\ v' \models \mathrm{inv}(\ell') \end{cases}$

delay transition: $(\ell, v) \xrightarrow{d} (\ell, v + d) \iff d \in \mathbb{R}_{\geq 0} \wedge v + d \models \mathrm{inv}(\ell)$

Time Petri Nets

Back

A *Time Petri Net* $\mathcal{N}$ is a tuple $(P, T, \Sigma_\varepsilon, {}^\bullet(.), (.)^\bullet, M_0, \Lambda, I)$ where:

- $P = \{p_1, p_2, \cdots, p_m\}$ is a finite set of places
- $T = \{t_1, t_2, \cdots, t_n\}$ is a finite set of transitions and $P \cap T = \emptyset$;
- Σ is a finite set of actions
- ${}^\bullet(.) \in (\mathbb{N}^P)^T$ is the backward incidence mapping; $(.)^\bullet \in (\mathbb{N}^P)^T$ is the forward incidence mapping;
- $M_0 \in \mathbb{N}^P$ is the initial marking;
- $\Lambda : T \to \Sigma_\varepsilon$ is the labeling function;
- $I : T \to \mathcal{I}(\mathbb{Q}_{\geq 0})$ associates with each transition a firing interval;

Semantics of Time Petri Nets

◂ Back

Let $\mathcal{N} = (P, T, \Sigma_\varepsilon, {}^\bullet(.), (.)^\bullet, M_0, \Lambda, I)$ be a Time Petri Net.

A state (M, ν) of $\mathcal{N}$ is a pair with in $\mathbb{N}^P$ and $\nu \in \mathbb{R}_{\geq 0}^{Enabled(M)}$.
$ADM(\mathcal{N})$ is the set of states of $\mathcal{N}$.

The semantics of $\mathcal{N}$ is a Timed Transition System $S_\mathcal{N} = (Q, \{q_0\}, T, \rightarrow)$:

- $Q = ADM(\mathcal{N})$,
- $q_0 = (M_0, \mathbf{0})$, $F' = \{(M, \nu) \mid \; M \in F\}$ and
- $\longrightarrow \in Q \times (T \cup \mathbb{R}_{\geq 0}) \times Q$ is the transition relations:
  - the discrete transition relation is defined $\forall t \in T$ by:

$$(M,\nu) \xrightarrow{\Lambda(t)} (M',\nu') \textit{ iff } \begin{cases} t \in Enabled(M) \wedge M' = M - {}^\bullet t + t^\bullet \\ \nu(t) \in I(t), \\ \forall t \in \mathbb{R}_{\geq 0}^{Enabled(M')}, \nu'(t) = \begin{cases} 0 \textit{ if } \uparrow enabled(t', M, t), \\ \nu(t) \textit{ otherwise.} \end{cases} \end{cases}$$

Semantics of Time Petri Nets

◂ Back

Let $\mathcal{N} = (P, T, \Sigma_\varepsilon, {}^\bullet(.), (.)^\bullet, M_0, \Lambda, I)$ be a Time Petri Net.

A state (M, ν) of $\mathcal{N}$ is a pair with in $\mathbb{N}^P$ and $\nu \in \mathbb{R}_{\geq 0}^{Enabled(M)}$.
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- $Q = ADM(\mathcal{N})$,
- $q_0 = (M_0, \mathbf{0})$, $F' = \{(M, \nu) \mid M \in F\}$ and
- $\longrightarrow \in Q \times (T \cup \mathbb{R}_{\geq 0}) \times Q$ is the transition relations:
  - the discrete transition relation is defined $\forall t \in T$ by:
  - continuous transition relation is defined $\forall d \in \mathbb{R}_{\geq 0}$ by:

$$(M, \nu) \xrightarrow{d} (M, \nu') \text{ iff } \begin{cases} \nu' = \nu + d \\ \forall t \in Enabled(M), \nu'(t) \in I(t)^{\downarrow} \end{cases}$$