Symbolic Unfoldings for Networks of Timed Automata

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Automated Technology for Verification and Analysis (ATVA'06)
October 23–26th, 2006

Beijing, China

Outline of the talk

- Unfoldings for Network of Automata
- Symbolic Unfoldings for Network of Timed Automata
- Conclusion

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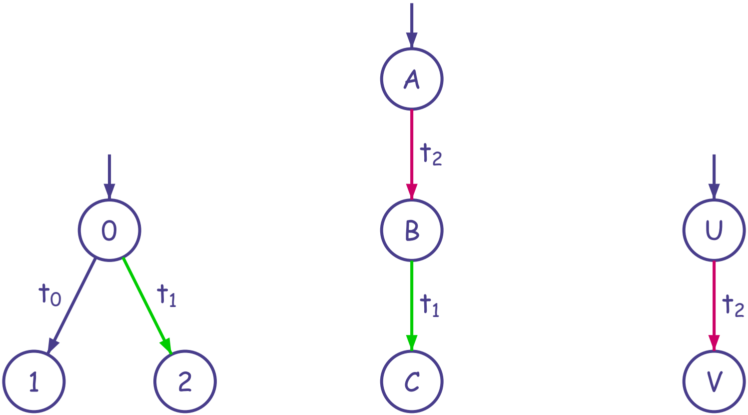
Unfoldings à la McMillan

For Petri Nets [McMillan, FMSD'95]
For Network of Automata [Esparza & Römer, CONCUR'99]



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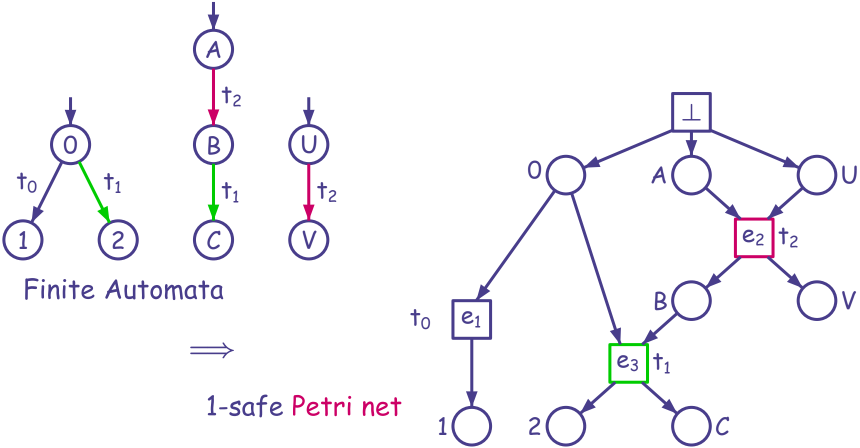
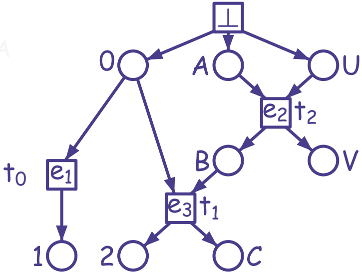


Features of Unfoldings

- Unfolding = 1-safe Petri net
- Finite "good" unfoldings exist
  finite complete prefix
- Preserves concurrency
  size(unfolding) < synchronous product of TA
- Can be constructed efficiently
- Can be used for checking properties:
  - coverability or reachability properties
  - deadlock detection
  - temporal logics properties
- Can be used for diagnosis:
  - Induces a partial order on events
  - Event structure = explanations for set of events



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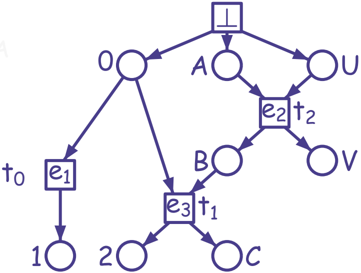

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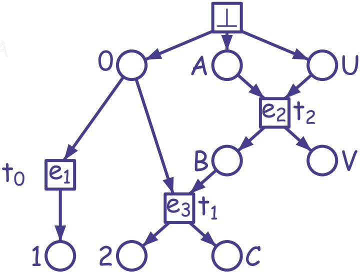

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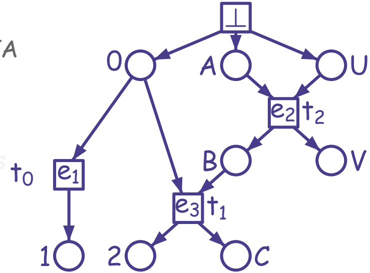

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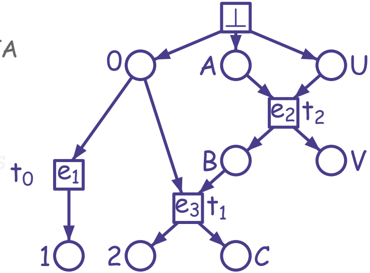

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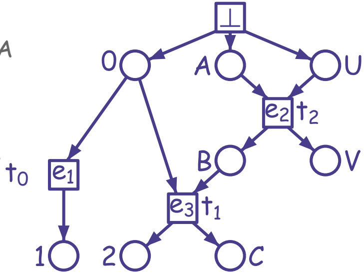

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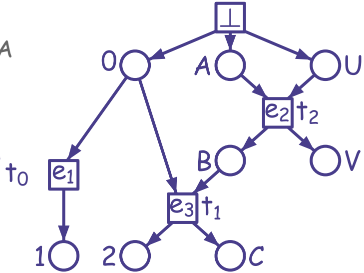
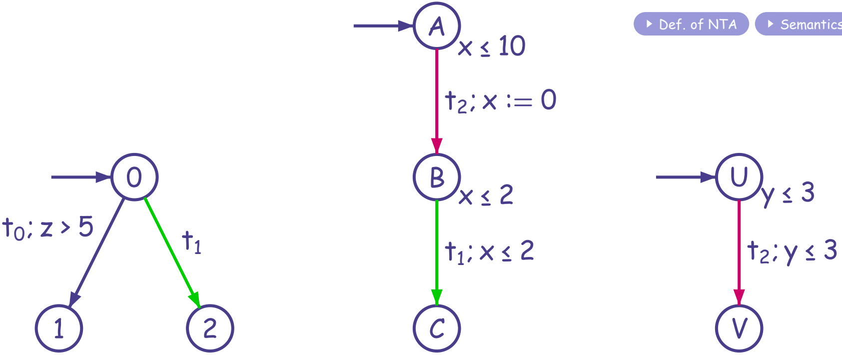


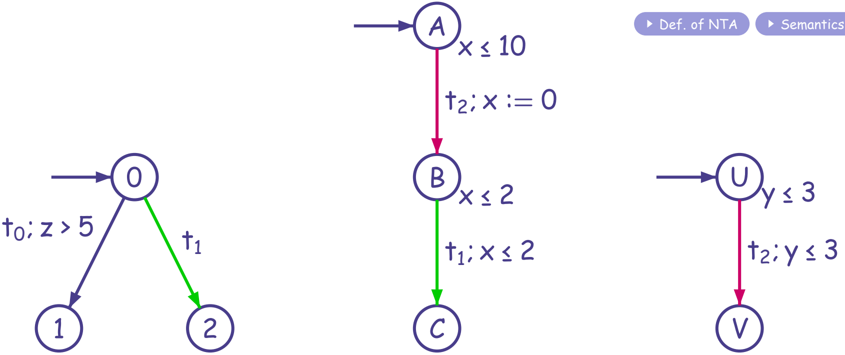
Network of Timed Automata

▸ Def. of NTA ▸ Semantics of NTA



Clocks are NOT shared

Network of Timed Automata



Def. of NTA
Semantics of NTA

State of a NTA: $((1, A, U), x = 1, y = 1, z = 1)$
Symbolic state: $((1, A, U), x = y = z \wedge y \leq 3)$

Clocks are NOT shared

Network of Timed Automata

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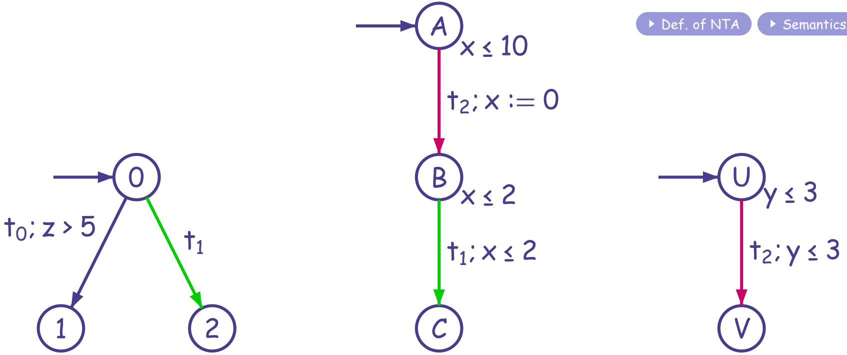

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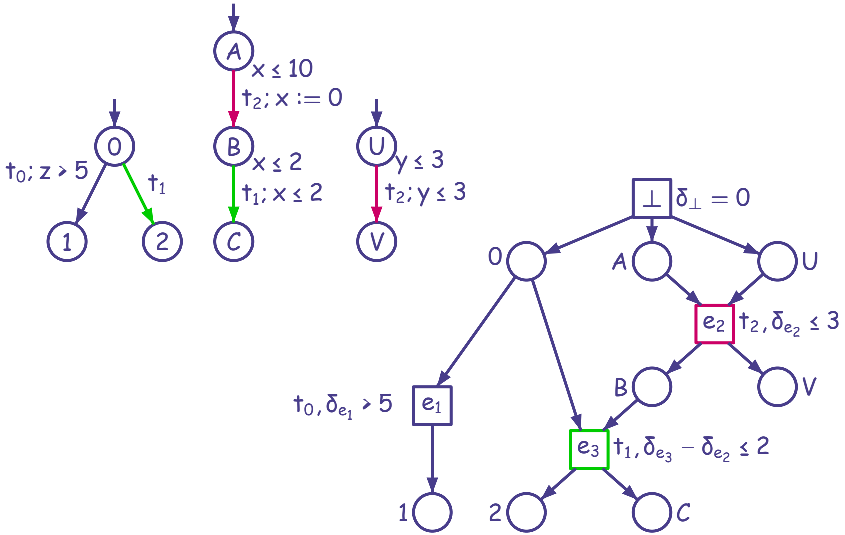
Unfoldings for Network of Timed Automata ?



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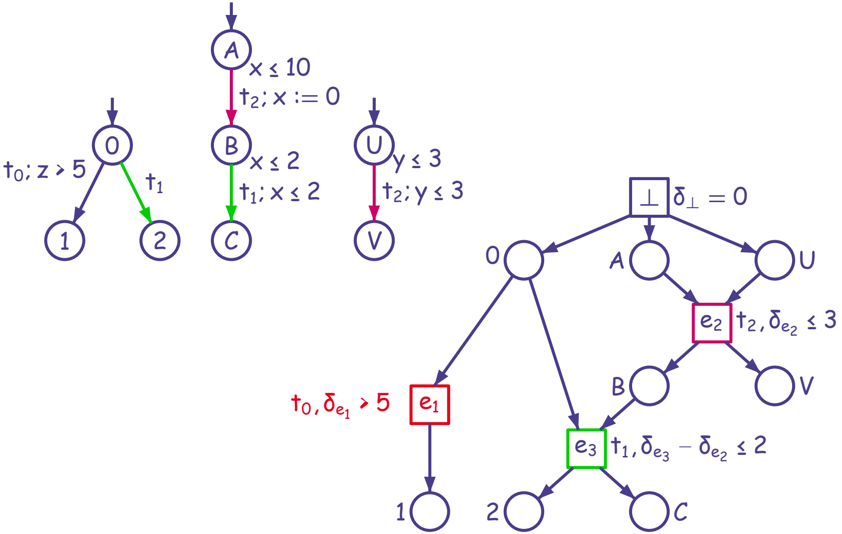

Related Work

- Unfoldings for Network of Timed Automata (NTA)
  - [Bengtsson et al., CONCUR'99, Minea, CONCUR'99]:
    semantics of NTA based on local time elapsing, assumption of time-stop freeness
  - [Lugiez et al., TACAS'04]:
    independence between transitions
    symbolic states have more clocks than the NTA
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    Applied to efficient model-checking of Timed Automata
- Unfoldings for Time Petri Nets (TPNs)
  - [Aura-Lilius, TCS'00]: Process Semantics for TPNs
    check realizability of a timed configuration
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Objectives & Results of this Paper

- Our Main Goal: give a concurrent semantics for NTA
  - Model for a concurrent semantics for timed systems
  - Define what is the concurrent semantics of a NTA
  - Finite representation
- Requirements for the concurrent semantics:
  - Preserves the concurrency of the system
  - Can be constructed efficiently
  - Allows to check basic properties (e.g. reachability)
- Results:
  - Model: 1-safe Petri nets with read arcs and timing information
    Symbolic Unfolding
  - An algorithm to compute a symbolic unfolding of a NTA
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- **Symbolic Unfoldings for Network of Timed Automata**
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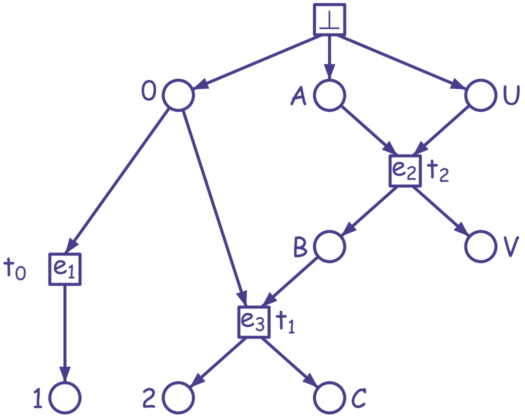
Basics: Configurations, Co-sets, Cuts

- Configuration: feasible set of events; past-closed
- Co-set: feasible set of places
- Cut: maximal co-set



Symbolic Unfolding - Step 1

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1. Build the unfolding of the underlying untimed network

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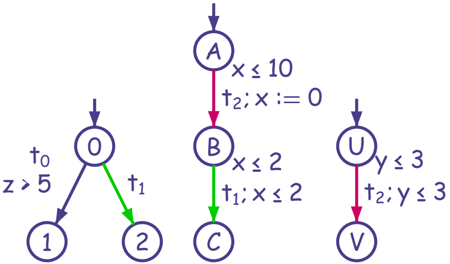
2. Annotate it with timing constraints on events

Symbolic Unfolding - Step 1



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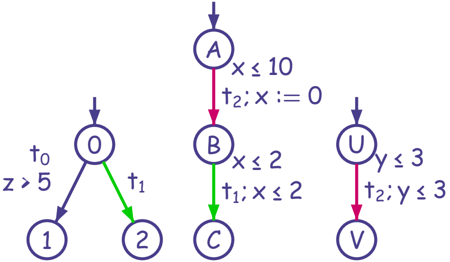

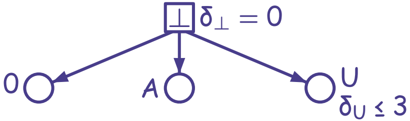

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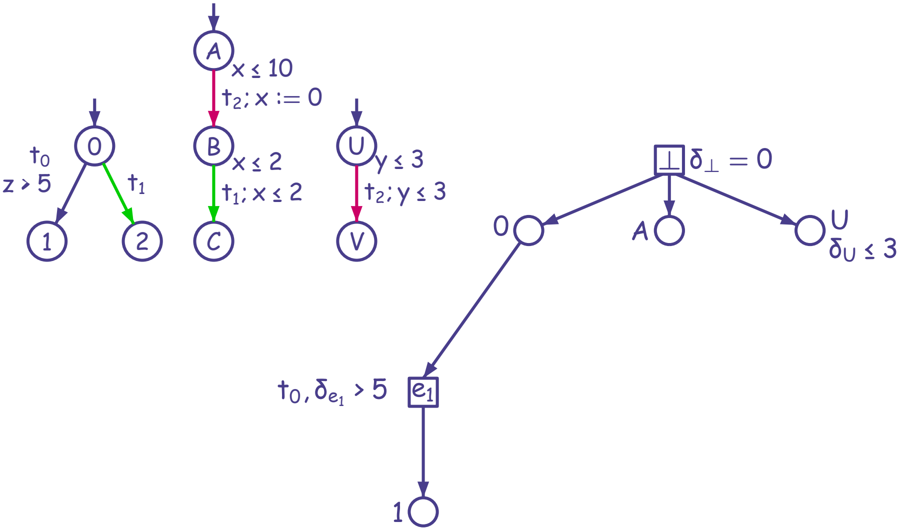

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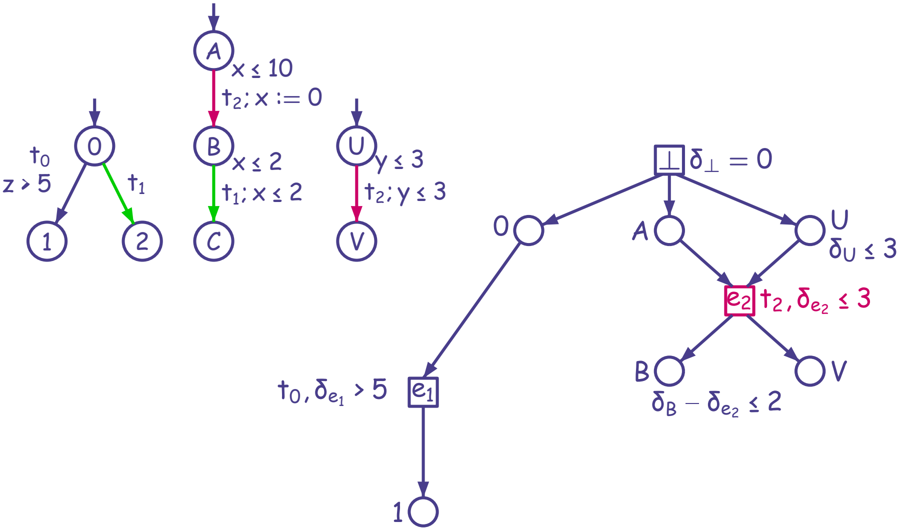

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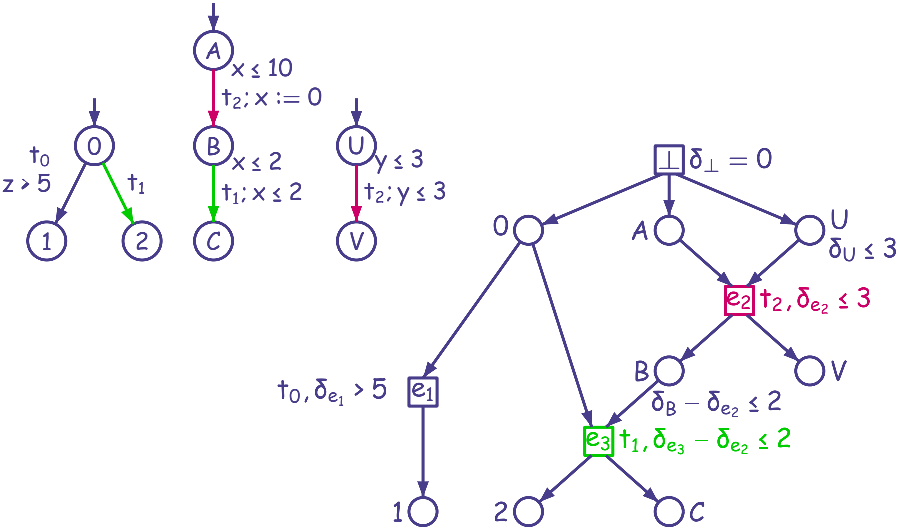

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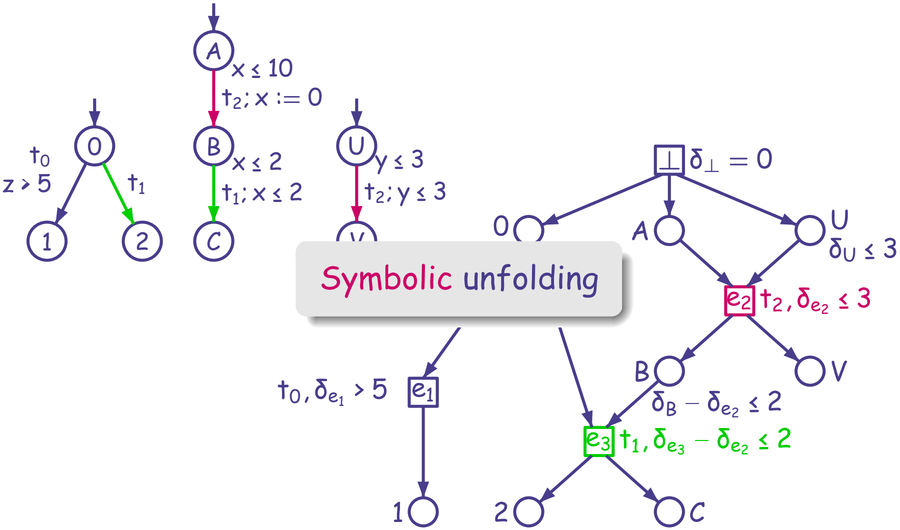

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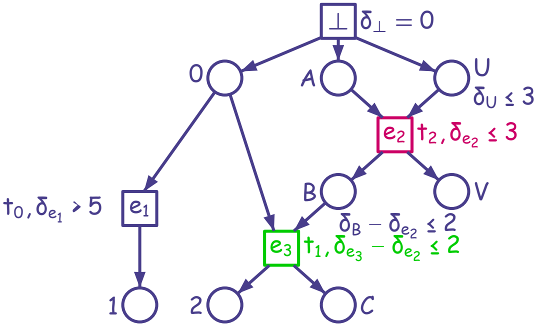

- Symbolic Cuts: $(C, \Phi(C))$
  $\Phi(C)$ is a constraint on the global time δ at which tokens can be in C

Def. of Symbolic Cuts

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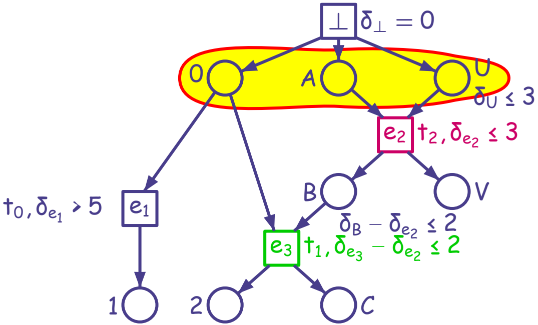

- Symbolic Cuts: $(C, \Phi(C))$ ▸ Def. of Symbolic Cuts
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$$(O, A, U), \delta \leq 3 \qquad (\delta = \delta_O = \delta_A = \delta_U \wedge \delta_U \leq 3)$$

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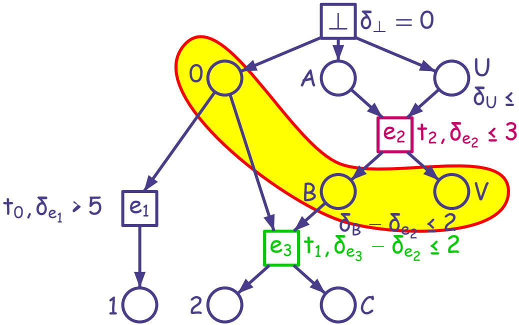
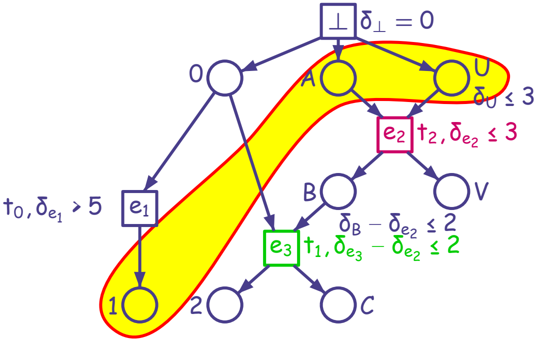


- **Symbolic Cuts**: $(C, \Phi(C))$
  $\Phi(C)$ is a constraint on the global time δ at which tokens can be in C

Def. of Symbolic Cuts

$$(O, B, V), \delta \geq \delta_{e_2} \wedge \delta - \delta_{e_2} \leq 2 \wedge \delta_{e_2} \leq 3$$

Properties of the Symbolic Unfolding



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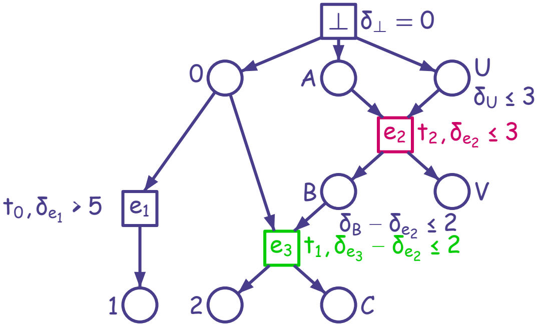
Def. of Symbolic Cuts

$$(1, A, U), \delta \geq \delta_{e_1} \wedge \delta_{e_1} > 5 \wedge \delta \leq 3$$

Properties of the Symbolic Unfolding



- one-to-one mapping f:
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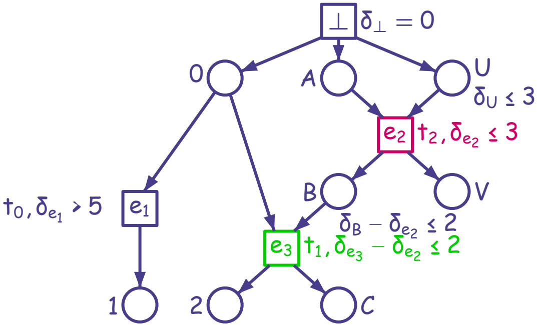

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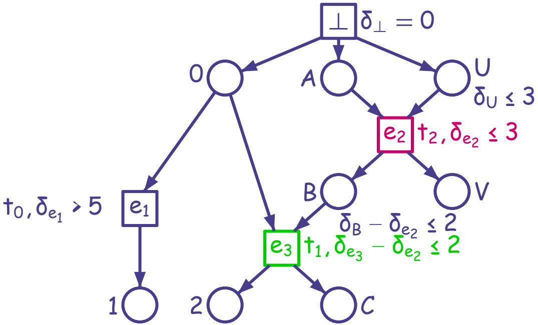
Theorem

For each cut C, $\Phi(C)$ is a zone. f preserves zones. $\cup_p(\vec{C}, Z_p)$ is a zone.

Gives an alternative proof of the result in **[Ben Salah, CONCUR'06]**

Properties of the Symbolic Unfolding



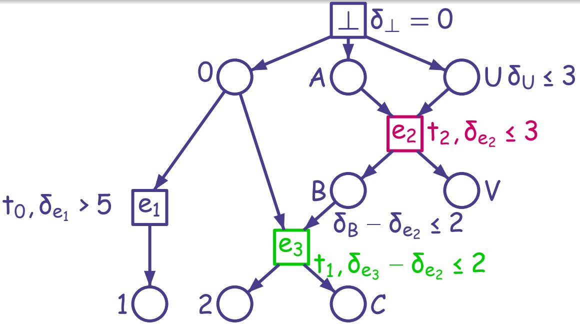
Theorem

For each cut C, $\Phi(C)$ is a zone. f preserves zones. $\cup_p(\vec{C}, Z_p)$ is a zone.

Theorem (Finite Complete Prefix)

Finite Complete prefixes exist for Network of Timed Automata.

Properties of the Symbolic Unfolding (Cont'd)



- Allows to check (non) emptyness of a symbolic cut: $[\![\Phi(C)]\!] \neq \emptyset$
  And thus check that a set of events can be extended to a configuration sub-configuration
  $\Longrightarrow$ try all the possible cuts ... not efficient
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How to check directly that a set of events is a sub-configuration ?

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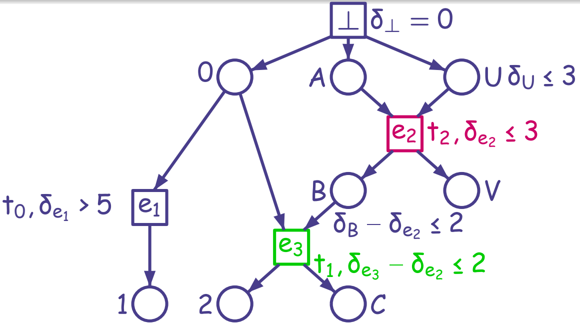

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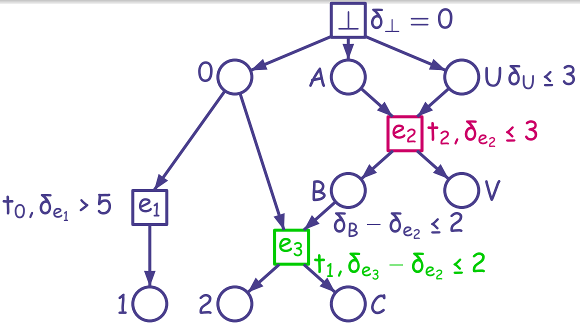

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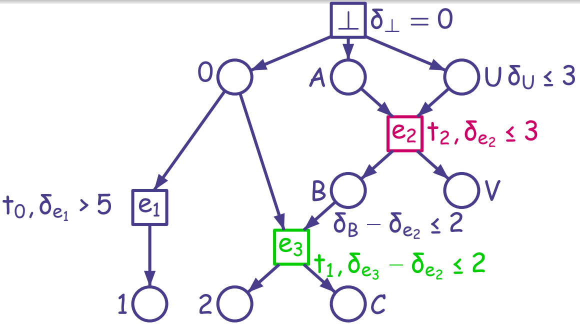

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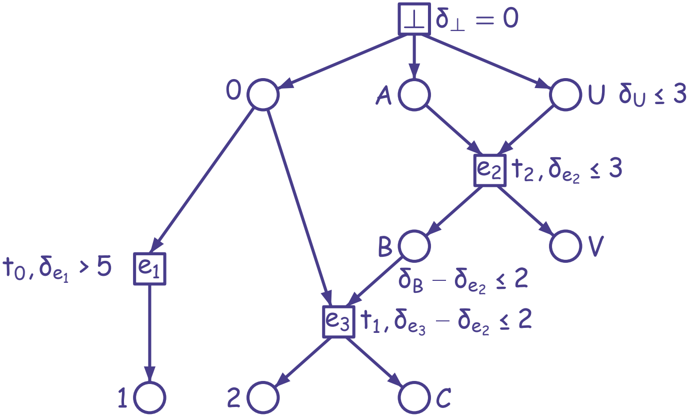
Symbolic Unfoldings – Step 2

- constraints on firing e_1 depends on the cuts that enable e_1



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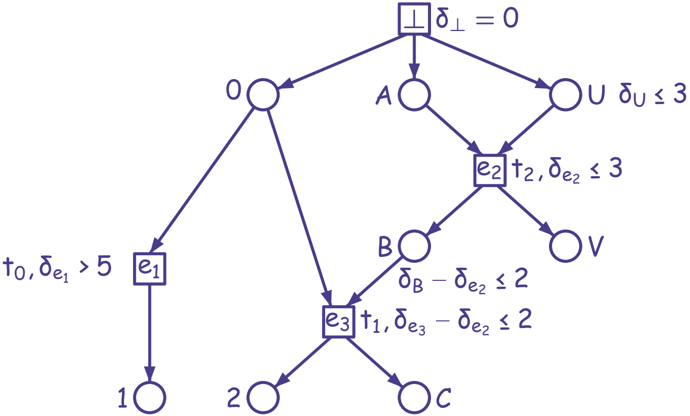

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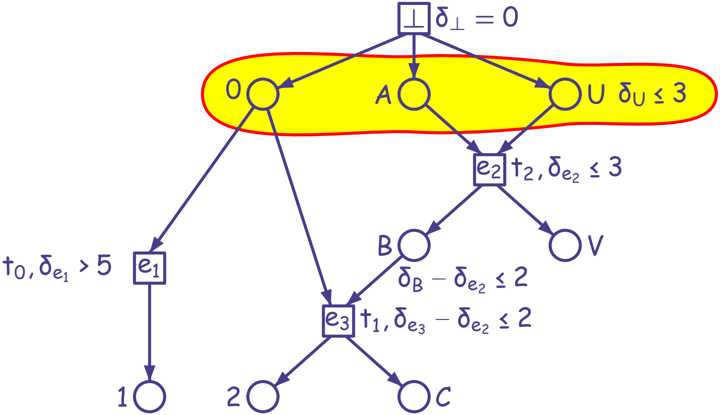

Symbolic Unfoldings – Step 2

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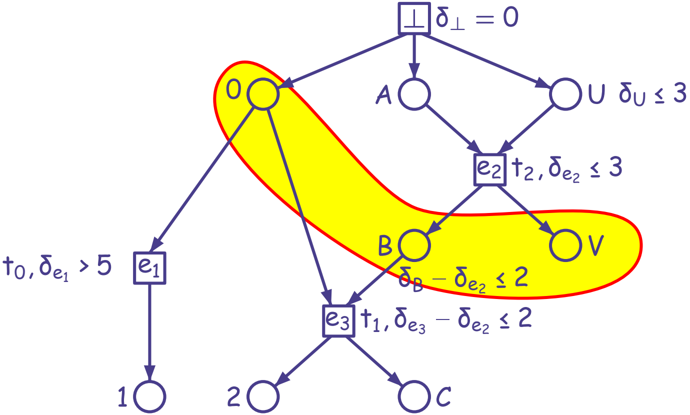
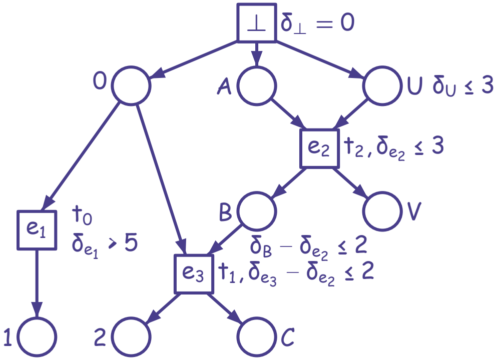
  for $(0, B, V)$: $\delta_{e_1} - \delta_{e_2} \leq 2$. $(\delta_{e_1} = \delta_0 = \delta_B = \delta_V \wedge \delta_B - \delta_{e_2} \leq 2)$



From Symbolic Unfolding to Extended Unfolding

- for each event e of the symbolic unfolding:
  1. $C(e)$ = set of enabling cuts of e
  2. compute the set of constraints generated by $C(e)$ on the (global) firing time δ of e



$$C(e_1) = \{(0, A, U), (0, B, V)\}$$
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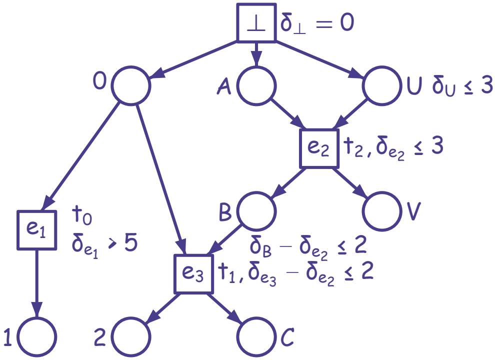

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- $(1, A, U)$: $\delta \leq 3$
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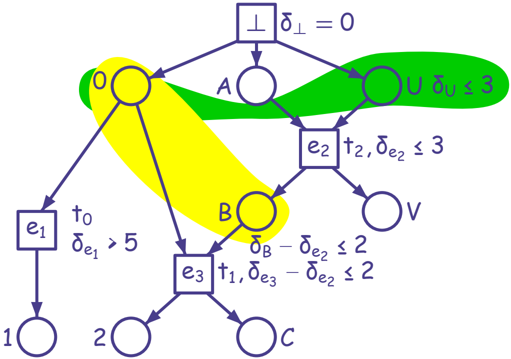

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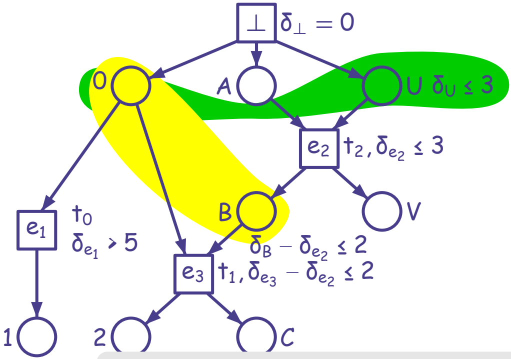

$C(e_1) = \{(O, A, U), (O, B, V)\}$

- (O, B, V):
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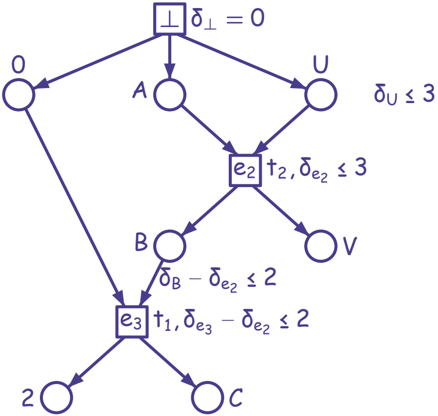
For $C(e_1)$, (O, B) and (O, U) generates the good constraints

(O, B) and (O, U) are Safe Representatives for event e_1

Extended Symbolic Unfolding

- to add an event e to a prefix:
  1. find a co-set C containing $^\bullet e$ and extend it to a safe representative S
     for e_1: $(0, U)$ and $(0, B)$
  2. use normal arcs from C to e and read arcs from $S \setminus C$ to e



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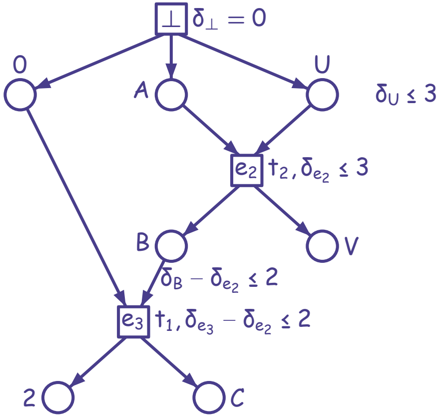

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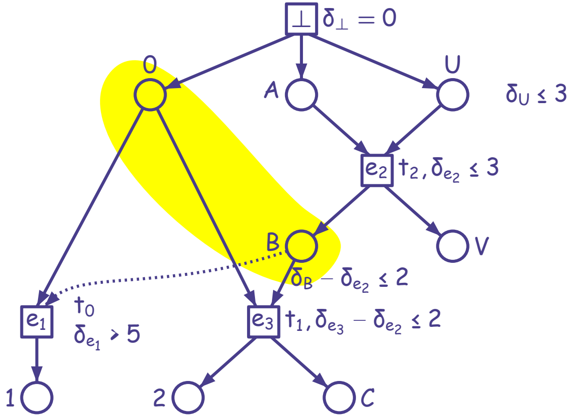

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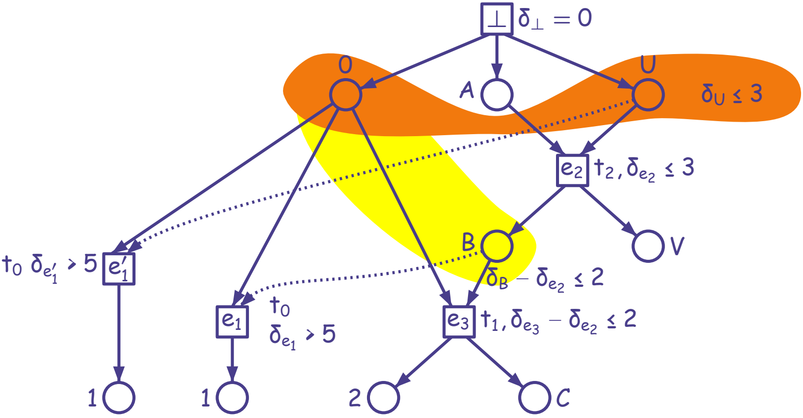

Properties of the Extended Unfolding

- Complete and Finite Extended Prefixes exists (not unique) even for NTA with "loops"
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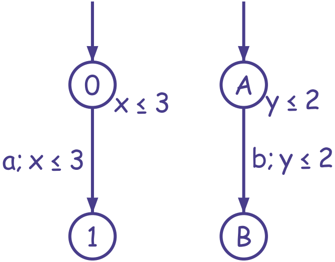
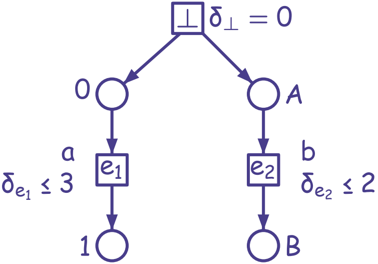


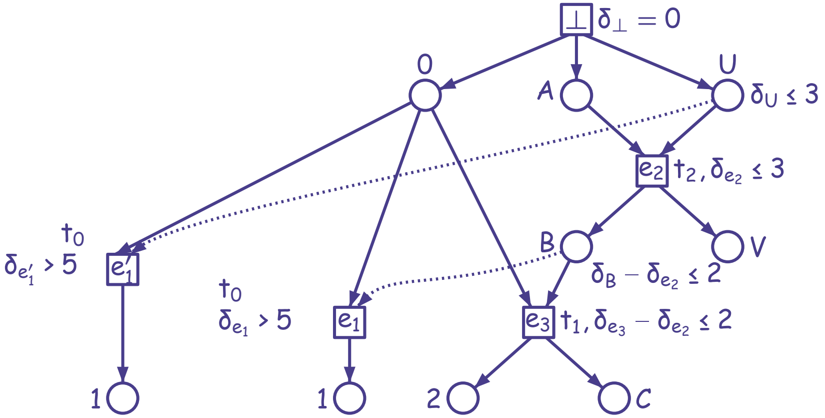
(a) Two Independent Automata



(b) The Unfolding

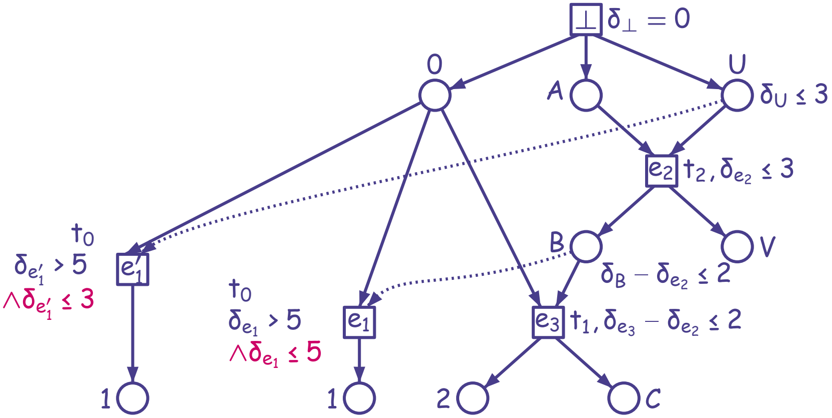
Properties of the Extended Unfolding (Cont'd)

- We can check **directly** that a set of **timed** events can be extended to a **(timed) configuration**



Properties of the Extended Unfolding (Cont'd)

- We can check directly that a set of timed events can be extended to a (timed) configuration



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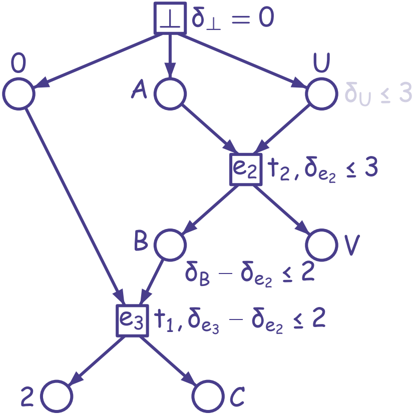

Outline

- Unfoldings for Network of Automata
- Symbolic Unfoldings for Network of Timed Automata
- **Conclusion**

Conclusion & Future Work

- Results:
  - Extended unfoldings for network of TA
    Petri nets with read arcs and timing constraints
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  - Unfolding preserves concurrency
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  - Induces a partial order of timed events
- Future Work:
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Timed Automata [Alur & Dill, TCS'94]

A Timed Automaton $\mathcal{A}$ is a tuple $(L, \ell_0, \Sigma, X, Inv, \longrightarrow)$ where:

- L is a finite set of locations
- ℓ_0 is the initial location
- X is a finite set of clocks
- Σ is a finite set of actions
- $\longrightarrow$ is a set of transitions of the form $\ell \xrightarrow{g,a,R} \ell'$ with:
  - $\ell, \ell' \in L$,
  - $a \in \Sigma$
  - a guard g which is a clock constraint over X
  - a reset set R which is the set of clocks to be reset to 0

Clock constraints are boolean combinations of $x \sim k$ with $x \in C$ and $k \in \mathbb{Z}$ and $\sim \in \{\leq, <\}$.

Back

Semantics of Timed Automata

Let $\mathcal{A} = (L, \ell_0, \Sigma, X, \mathrm{Inv}, \longrightarrow)$ be a Timed Automaton.

A **state** (ℓ, v) of $\mathcal{A}$ is in $L \times \mathbb{R}_{\geq 0}^X$

The semantics of $\mathcal{A}$ is a **Timed Transition System** $S_\mathcal{A} = (Q, q_0, \Sigma \cup \mathbb{R}_{\geq 0}, \longrightarrow)$ with:

- $Q = L \times \mathbb{R}_{\geq 0}^X$
- $q_0 = (\ell_0, \overline{0})$
- $\longrightarrow$ consists in:

discrete transition: $(\ell, v) \xrightarrow{a} (\ell', v') \iff \begin{cases} \exists\, \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A} \\ v \models g \\ v' = v[r \leftarrow 0] \\ v' \models \mathrm{Inv}(\ell') \end{cases}$

delay transition: $(\ell, v) \xrightarrow{d} (\ell, v + d) \iff d \in \mathbb{R}_{\geq 0} \wedge v + d \models \mathrm{Inv}(\ell)$

Back

Network of Timed Automata

Back

Let $\mathcal{A}_i = (L_i, \ell_0^i, \Sigma_i, X_i, Inv_i, \longrightarrow_i)$ be Timed Automata.

$L_i \cap L_j = \emptyset$ and $X_i \cap X_j = \emptyset$ $\quad$ $L = \cup_i L_i$ and $X = \cup_i X_i$

A state $(\vec{\ell}, \vec{v})$ of $\mathcal{A}$ is in $L \times \mathbb{R}_{\geq 0}^X$.

Assume each TA has a loop transition $(\ell, true, \varepsilon, \emptyset, \ell)$.

I is the synchonization function.

The network $\mathcal{A} = (\mathcal{A}_1 \times \cdots \times \mathcal{A}_n)_I$ is defined by:

- $Q = L \times \mathbb{R}_{\geq 0}^X$
- $q_0 = (\ell_0, \overline{0})$ with $\ell_0 = (\ell_0^1, \cdots, \ell_0^n)$
- $\longrightarrow$ consists in:

discrete transition: $$(\vec{\ell}, v) \xrightarrow{a} (\vec{\ell}', v') \iff \begin{cases} \exists a = (a_1, \cdots, a_n) \in I \\ \exists\ \ell_i \xrightarrow{g_i, a_i, r_i}_i \ell_i' \in \mathcal{A}_i \\ v \models \wedge_i g_i \\ v' = v[\cup_i v_i \leftarrow 0] \\ v' \models \wedge_i Inv_i(\ell_i') \end{cases}$$

delay transition: $$(\vec{\ell}, v) \xrightarrow{d} (\vec{\ell}, v + d) \iff d \in \mathbb{R}_{\geq 0} \wedge v + d \models \wedge_i Inv_i(\ell_i)$$

Symbolic (or Timed) Cuts

Back

$(C, \Phi(C))$ is a symbolic cut if:

1. C is a untimed cut
2. $\Phi(C) = \Phi_1(C) \wedge \Phi_2(C) \wedge \Phi_3(C) \wedge \Phi_4(C)$ where $\Phi_i(C), 1 \leq i \leq 4$ are defined by:

$$\Phi_1(C) = \bigwedge_{x \in \lceil C \rceil} \gamma(x) \quad (1) \qquad \Phi_3(C) = \bigwedge_{p \in C} (\delta_{\bullet p} \leq \delta_p) \quad (3)$$

$$\Phi_2(C) = \bigwedge_{e \in \lceil C \rceil \cap E} (\wedge_{p \in {}^\bullet e} \delta_p = \delta_e) \quad (2) \qquad \Phi_4(C) = (\bigwedge_{p,p' \in C} \delta_p = \delta_{p'}) \quad (4)$$

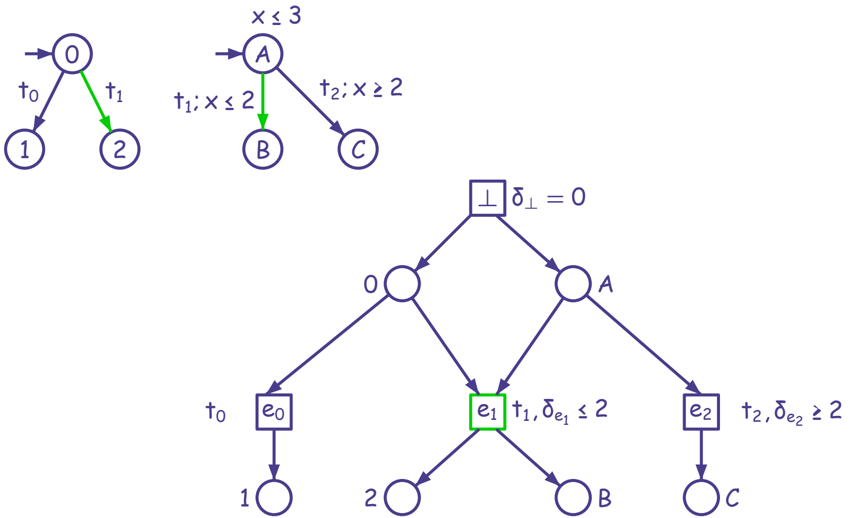
with $\gamma(x)$ the constraint associated with node x.

Let G be the simulation graph of the network of TA and $\mathcal{N}$ be a symbolic unfolding of the NTA

Theorem

$(C, \Phi(C))$ is a symbolic cut of $\mathcal{N}$ and $[\![\Phi(C)]\!] \neq \emptyset$ iff C is reachable in G.

Time Constraints on Places



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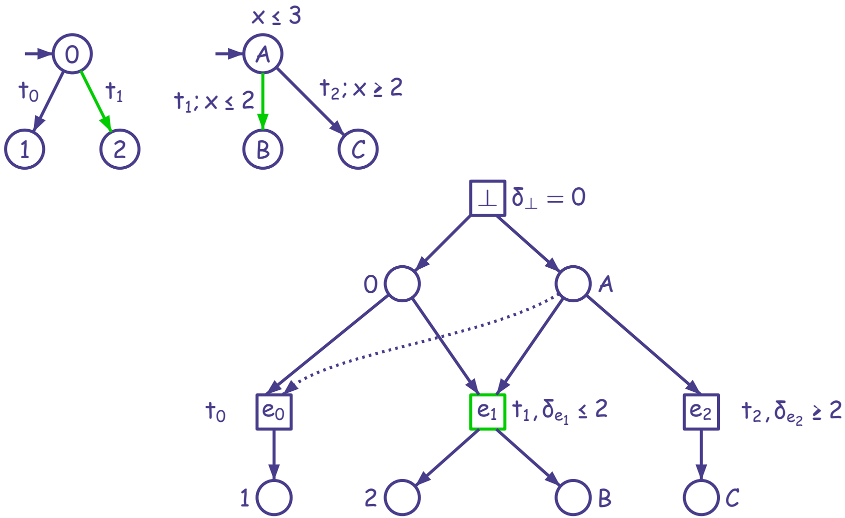

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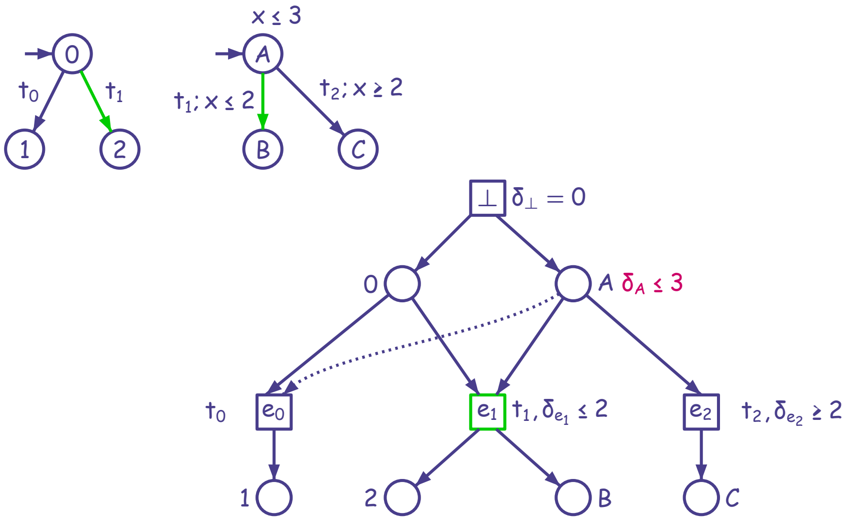

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