Monitoring and Fault-Diagnosis with Digital Clocks

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Motivation & Context

Monitoring

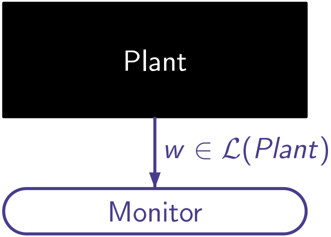
Plant generates $\mathcal{L}(Plant) \subseteq \Sigma^*$

Specification $= \mathcal{L}(S) \subseteq \Sigma^*$

Diagnosis

Plant generates $\mathcal{L}(Plant) \subseteq (\Sigma \cup \{\varepsilon, f\})^*$

Spec. $= \mathcal{L}(S) = \{\rho.f.\rho' \text{ s.t. } |\rho'| \geq k\}$



Role of the monitor:

- can shout when $w \notin \mathcal{L}(S)$
- never shout when $w \in \mathcal{L}(S)$

Role of the k-diagnoser:

- must shout when $w \in \mathcal{L}(S)$
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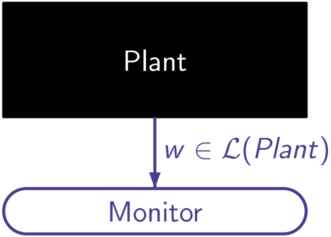

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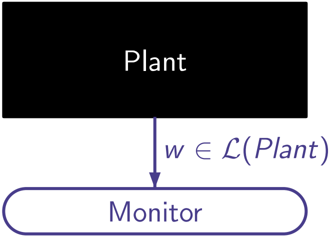

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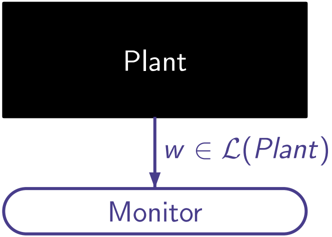

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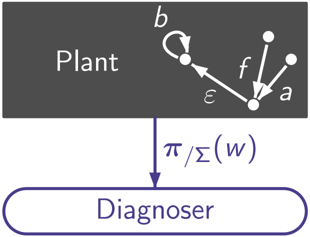

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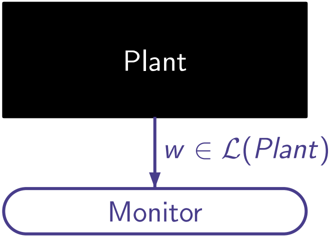

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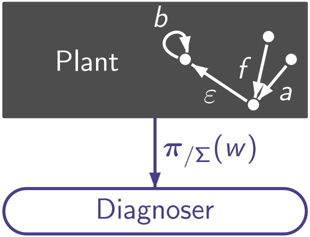

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Known Results & Related Work

- Discrete Events Systems [Sampath et al., IEEE'95] — Finite Automata
  - Monitoring ≡ determinize the specification
  - Diagnosis
    1. Check diagnosability (PTIME)
    2. Compute a diagnoser (EXPTIME)
- Dense-time Systems — Timed Automata
  - Monitoring
    TA are not determinizable – Checking determinizability is undecidable
    On-the-fly solutions [Krichen, Tripakis, FORMATS'04]
  - Diagnosis
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Use of Analog Clocks = arbitrarily precise

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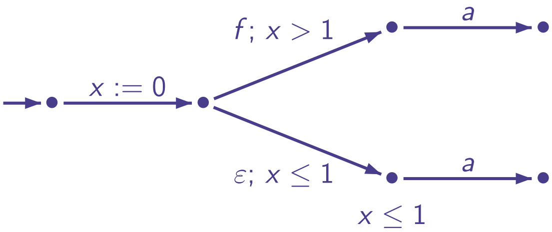
Our contribution: Monitoring & Fault Diagnosis with Digital Clocks

Perfect Clocks vs. Fuzzy Clocks

Digital Clocks cannot have arbitrary precision: imprecision Δ



Perfect Clock t: if $a@t$ and $t > 1$ say "Fault" otherwise say nothing

Fuzzy Clocks: value of t is an interval $[t - \Delta, t + \Delta]$

$f@(1 + \frac{\Delta}{4}).a@(1 + \frac{\Delta}{3})$ and $\varepsilon@1.a@(1 + \frac{\Delta}{2})$ are indistinguishable

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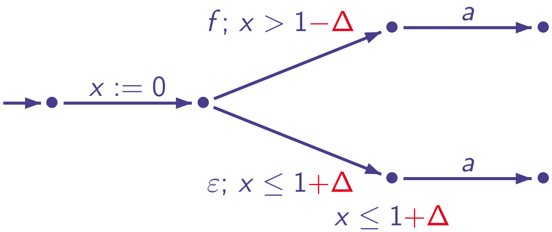

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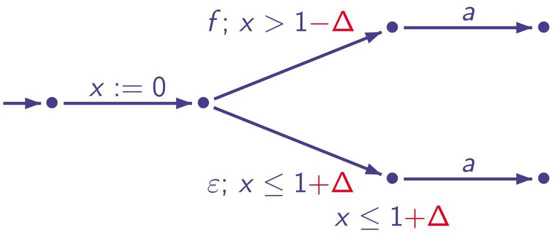

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- Models for Timed Systems & Digital Clocks
- Monitoring with Digital Clocks
- Diagnosis with Digital Clocks
- Conclusion & Open Problem

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Timed Automata

[Alur & Dill, TCS'94]

- Timed Automaton = Finite Automaton + clock variables
  All clocks evolve at the same speed
  Clocks take their value in a dense-time domain
- Transitions are guarded by clocks constraints
- g: guard of the form $g ::= x \sim c \,|\, g \wedge g$
  where x is a clock and $c \in \mathbb{N}$, $\sim \in \{<, \leq, =, \geq, >\}$
- R : the set of clocks to be reset when firing the transition
- $\mathrm{Inv}(\ell)$ is an invariant to ensure "liveness"
- Semantics of TA: Timed Transition Systems

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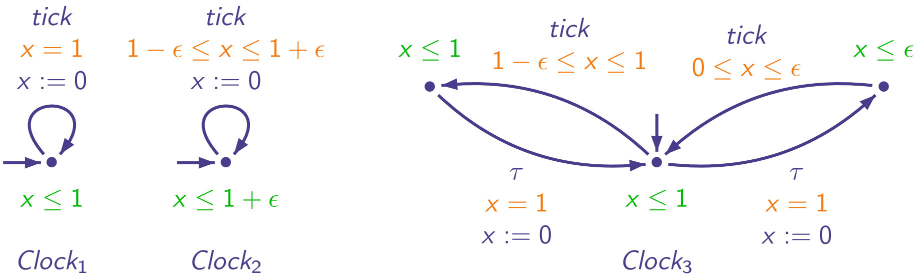

Timed Words:

$Clock_1$: $1.tick.1.tick.\cdots.1.tick.\cdots$

Let $\epsilon = 0,3$

$Clock_2$: $0,8.tick.1,14.tick.\cdots.0,98.tick.\cdots$

n^{th} tick at t with $n \cdot (1 - 0,3) \leq t \leq n \cdot (1 + 0,3)$

$Clock_3$: $0,8.tick.1,3.tick.\cdots.1,15.tick.\cdots$

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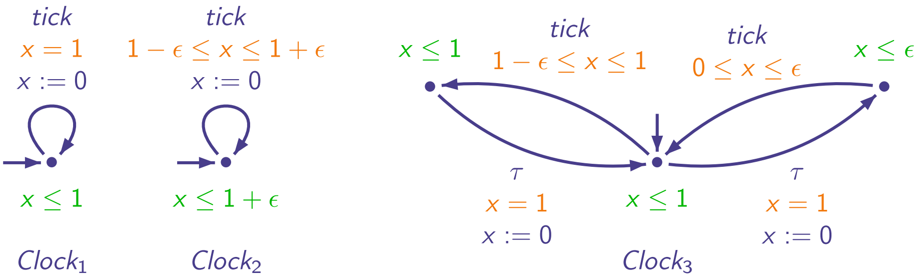

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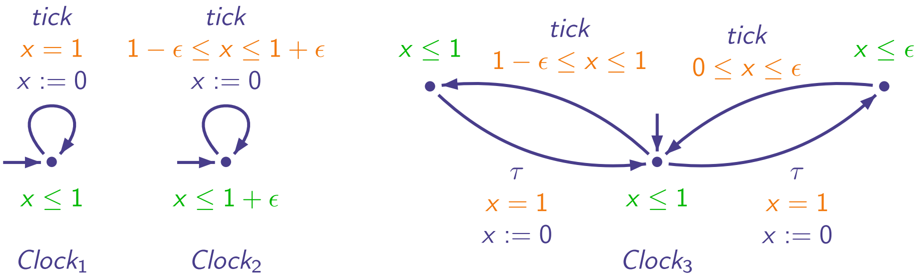

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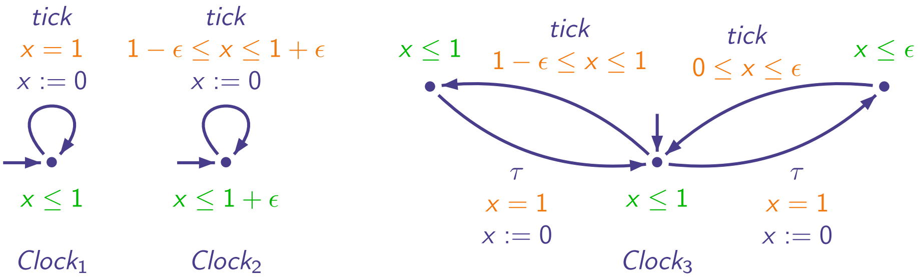

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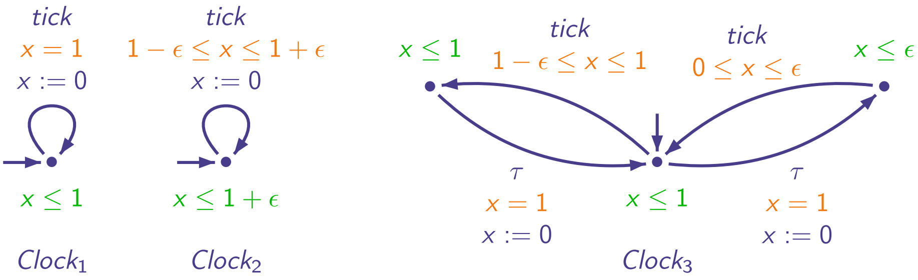

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Timed Languages & Region Graph/Automaton

- Timed words: alternating sequences of symbols in $\Sigma \cup \mathbb{R}_{\geq 0}$
  Dense-Time: $0.a.\pi.b.\frac{1}{3}.b.\cdots$
  $1.a.2.\varepsilon.1.b = 1.a.3.b$
- Timed Language = set of timed words accepted by a timed automaton
  $\mathcal{L}(A)$ and $\mathcal{L}^\omega(A)$
- Untimed Language = projection on Σ of the Timed Language
  $\pi_{/\Sigma}(1.a.2.\varepsilon.1.b.1) = a.b$
  $\text{Duration}(1.a.2.\varepsilon.1.b.1) = 4$
- Product of timed words/languages: $w \parallel w'$ (for languages $L \parallel L'$)
  $1.a.2.b \parallel 0,5.c.1.d = 0,5.c.0,5.a.0,5.d.1,5.b$
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  Dense-Time: $0.a.\pi.b.\frac{1}{3}.b.\cdots$
  $1.a.2.\varepsilon.1.b \equiv 1.a.3.b$
- Timed Language = set of timed words accepted by a timed automaton
  $\mathcal{L}(A)$ and $\mathcal{L}^{\omega}(A)$
- Untimed Language = projection on Σ of the Timed Language
  $\pi_{/\Sigma}(1.a.2.\varepsilon.1.b.1) = a.b$
  $\text{Duration}(1.a.2.\varepsilon.1.b.1) = 4$
- Product of timed words/languages: $w \parallel w'$ (for languages $L \parallel L'$)
  $1.a.2.b \parallel 0,5.c.1.d = 0,5.c.0,5.a.0,5.d.1,5.b$
  $1.a \parallel 1.b = \{1.a.0.b, 1.b.0.a\}$
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Product of Automata

Given A and B, we can effectively build a TA $(A \parallel B)$ that accepts the timed language $\mathcal{L}(A) \parallel \mathcal{L}(B)$.

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Theorem (Region Graph ▸ Region Graph)

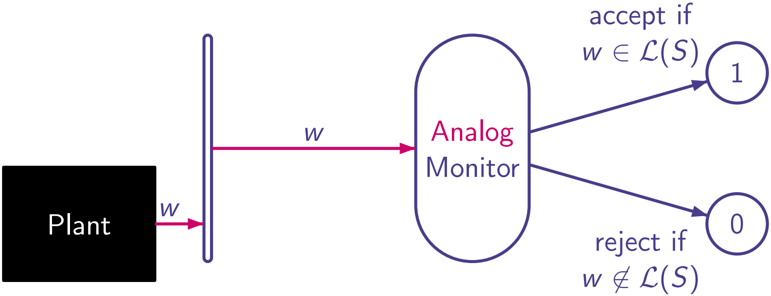
For each Timed Automaton A, we can effectively build a finite automaton $RG(A)$ s.t. $\mathcal{L}(RG(A)) = Untimed(\mathcal{L}(A))$. [Alur & Dill, TCS'94]

Outline

- Models for Timed Systems & Digital Clocks
- Monitoring with Digital Clocks
- Diagnosis with Digital Clocks
- Conclusion & Open Problem

Monitors & Digital Clocks

Specification: $\mathcal{L}(S)$



- Plant: generates timed words $w = t_0 a_0 t_1 a_1 \cdots t_n a_n$
- Digital Clock: generates $v \in (tick \cup \mathbb{R}_{\geq 0})^*$, non zeno
- Plant $\|$ Clock: generates timed words in $(\Sigma \cup \{tick\} \cup \mathbb{R}_{\geq 0})^*$

  $\rho = v \| w = 1.a.0.tick.2.b.1.tick.2.tick.8$
- Monitor: deterministic, accepts untimed words in $(\Sigma \cup \{tick\})^*$

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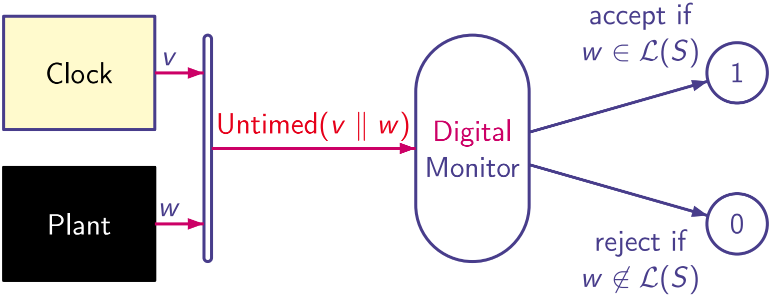

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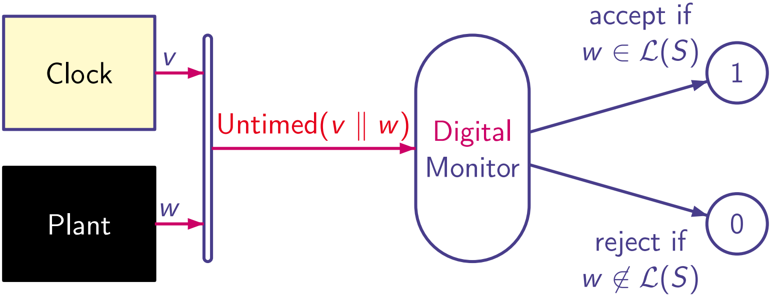

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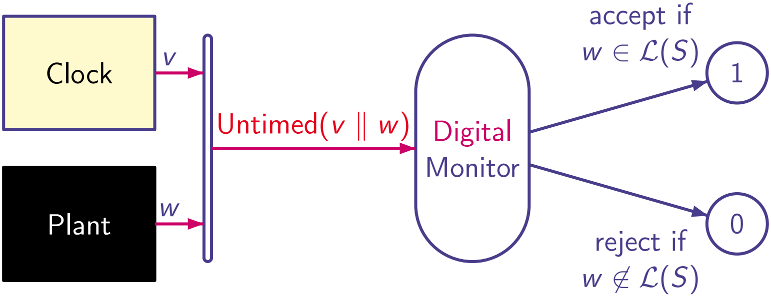

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Sound Monitors

Definition (Soundness)

An monitor M is sound w.r.t. $Clock$ if $\forall \rho \in \mathcal{L}(S)$ and $\rho' \in \mathcal{L}(Clock)$ M accepts $\text{Untimed}(\rho \parallel \rho')$ (or equivalently $M(\text{Untimed}(\rho \parallel \rho')) = 1$).

This is NOT equivalent to $\mathcal{L}(S) \subseteq (\mathcal{L}(M) \parallel \mathcal{L}(Clock))$

Property 1 (Better Clock Preserves Soundness)

If M is sound w.r.t. $Clock_1$ and $\mathcal{L}(Clock_2) \subseteq \mathcal{L}(Clock_1)$ then M is sound w.r.t. $Clock_2$.

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Construction of the Optimal Sound Monitor

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Inputs: Two timed automata S and $Clock$.

Problem: Build a sound monitor.

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Definition (Order on Monitors)

M is better than M' if $\mathcal{L}(M) \subseteq \mathcal{L}(M')$.

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Theorem (Soundess and Optimality of M_0)

M_0 is sound and optimal.

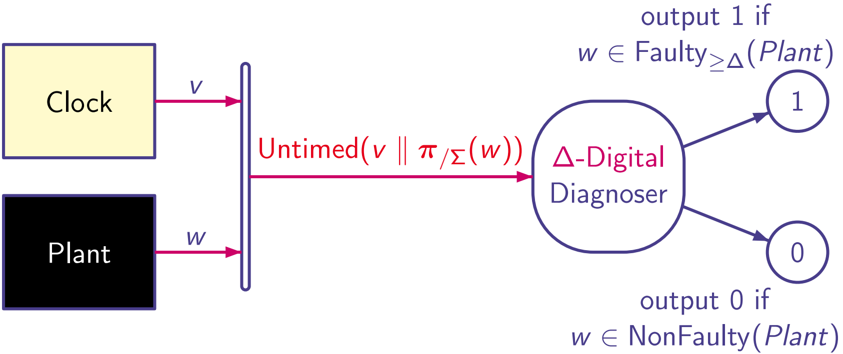
Proof

Soundness: If not, $\exists u \in \mathcal{L}(RG)$ s.t. $u \notin \text{Untimed}(\mathcal{L}(S) \parallel \mathcal{L}(Clock))$.
Optimality: By Property 2, a sound monitor must contain at least $\text{Untimed}(\mathcal{L}(S) \parallel \mathcal{L}(Clock))$ which is equal to $\mathcal{L}(RG)$.

Outline

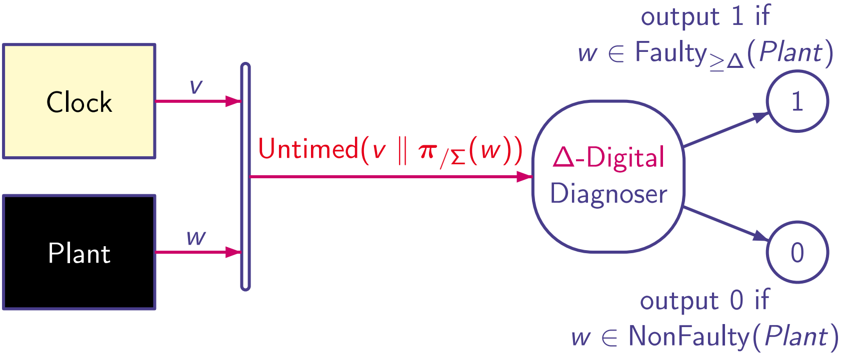
- Models for Timed Systems & Digital Clocks
- Monitoring with Digital Clocks
- **Diagnosis with Digital Clocks**
- Conclusion & Open Problem

Δ-Diagnosers & Digital Clocks



- Plant: ε and f unobservable
- $\rho = \rho_1.f.\rho_2$ is Δ-faulty if $f \notin \rho_1$ and $\text{Duration}(\rho_2) \geq \Delta$
  If $f \notin \rho$ then ρ is non faulty
- A Diagnoser D does not change its mind:
  $D(\rho) = 1 \implies D(\rho.\rho') = 1.$

Δ-Diagnosers & Digital Clocks



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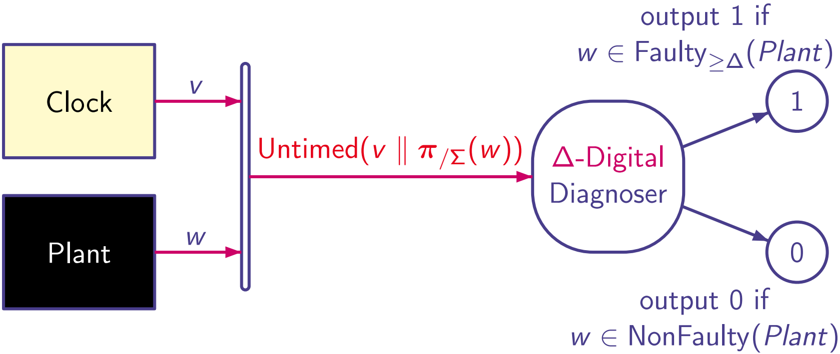

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Diagnosers & Diagnosability Problems

Definition ($(Clock, \Delta)$-Diagnosability)

$D : (\Sigma \cup \{tick\})^* \rightarrow \{0, 1\}$ is a $(Clock, \Delta)$-diagnoser for *Plant* if for any runs $\rho \in \mathcal{L}(Plant)$ and $\rho' \in \mathcal{L}(Clock)$ with $\text{Duration}(\rho) = \text{Duration}(\rho')$

- if $\rho \in \text{NonFaulty}(Plant)$ then $D(\text{Untimed}(\rho \parallel \rho')) = 0$
- if $\rho \in \text{Faulty}_{\geq\Delta}(Plant)$ then $D(\text{Untimed}(\rho \parallel \rho')) = 1$

Plant is $(Clock, \Delta)$-Diagnosable if $\exists$ a $(Clock, \Delta)$-diagnoser D.

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$Plant$ is ($Clock, \Delta$)-Diagnosable if $\exists$ a ($Clock, \Delta$)-diagnoser D.

Property 3 (Better Clocks . . .)

For any timed automata A, $Clock_1$ and $Clock_2$, for any $\Delta_1, \Delta_2 \in \mathbb{N}$, if D is a ($Clock_1, \Delta_1$)-diagnoser for A and $L(Clock_2) \subseteq L(Clock_1)$ and $\Delta_2 \geq \Delta_1$, then D is also a ($Clock_2, \Delta_2$)-diagnoser for A.

Diagnosers & Diagnosability Problems

Problem 2: $(Clock, \Delta)$-Diagnosability

Inputs: Two timed automata *Plant* and *Clock* and $\Delta \in \mathbb{N}$.
Problem: Check whether *Plant* is $(Clock, \Delta)$-diagnosable.

Diagnosers & Diagnosability Problems

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Problem 3: *Clock*-Diagnosability

Inputs: Two timed automata *Plant* and *Clock*.
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Problem 4: Diagnosability

Inputs: A timed automaton *Plant*.

Problem: Check whether $\exists$ a TA *Clock* s.t. *Plant* is *Clock*-diagnosable.

Solution to Problem 2

C_1: Necess. and Suffi. Condition for $(Clock, \Delta)$-diagnosability

Plant is $(Clock, \Delta)$-diagnosable iff $\forall \rho, \rho' \in \mathcal{L}(Plant)$, $\sigma, \sigma' \in \mathcal{L}(Clock)$

$$\left.\begin{array}{c} \rho \in \text{Faulty}_{\geq \Delta}(Plant) \\ \rho' \in \text{NonFaulty}(Plant) \\ \text{Duration}(\rho) = \text{Duration}(\sigma) \\ \text{Duration}(\rho') = \text{Duration}(\sigma') \end{array}\right\} \implies \text{Untimed}(\rho \parallel \sigma) \cap \text{Untimed}(\rho' \parallel \sigma') = \emptyset$$

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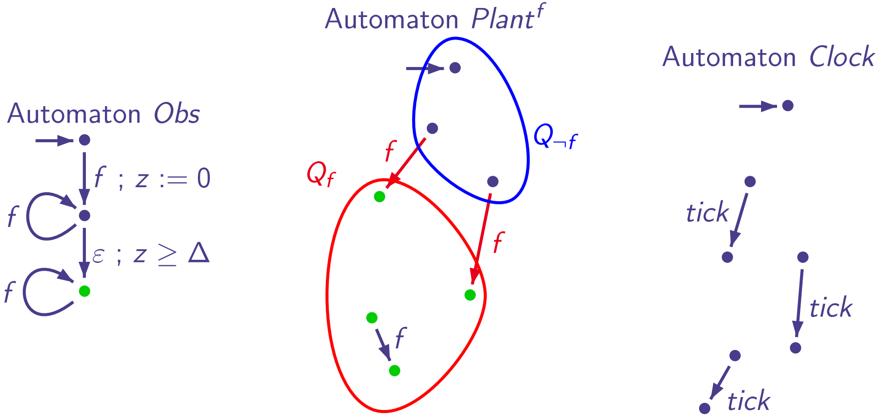

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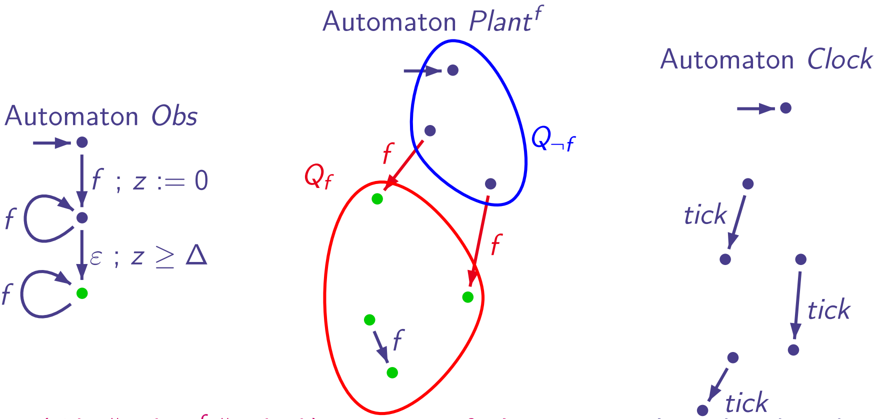

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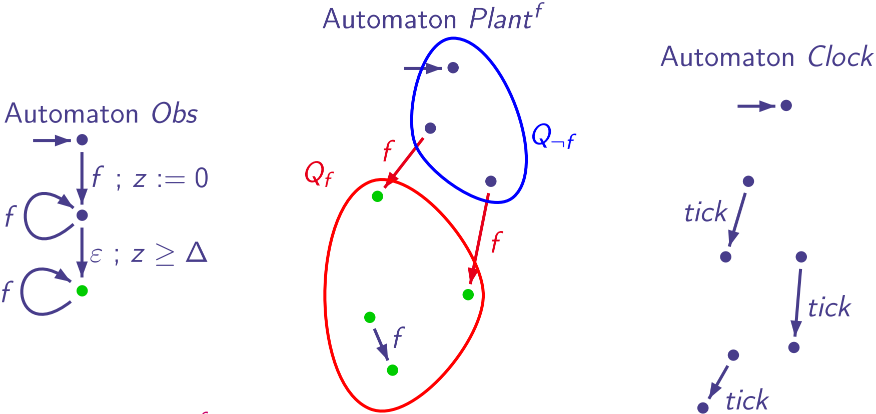

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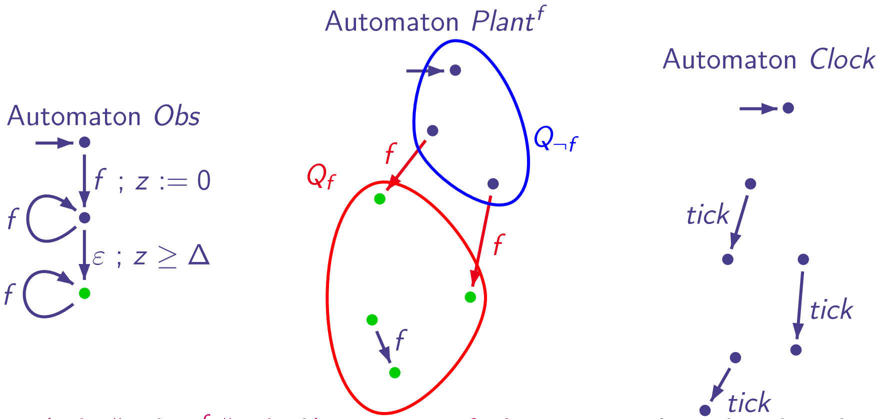

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Solution to Problem 3

Problem 3: Diagnosability

Inputs: Two timed automata *Plant* and *Clock*.
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For DES: amounts to checking (Büchi) emptyness

Algorithm for DES

Assumption: *Plant* is non zeno

C_2: Necess. and Suffi. Condition for *Clock*-diagnosability

Plant is NOT *Clock*-diagnosable iff $\exists \rho, \rho' \in \mathcal{L}^\omega(Plant), \sigma, \sigma' \in \mathcal{L}^\omega(Clock)$

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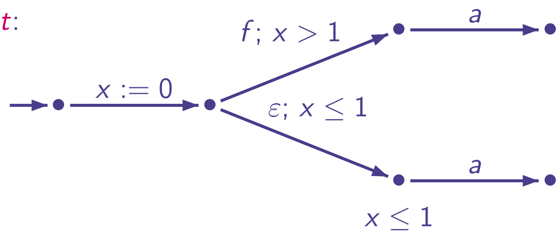

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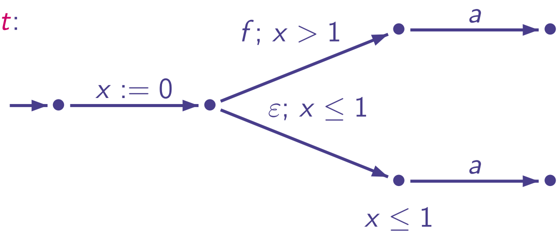
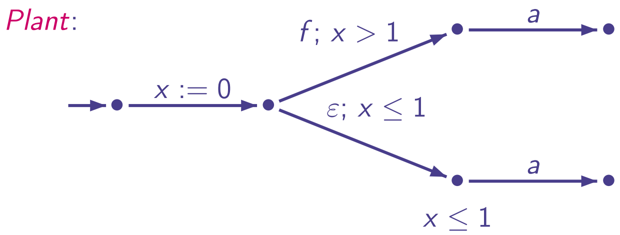


Plant is NOT *Clock*-diagnosable for any TA *Clock*.

Problem 4: Existence of a Digital Clock Diagnoser



Time: 1 t' 2

ticks: $\bullet^1$ $\cdots$ $\bullet$ $\bullet^n$ $\cdots$ $\bullet^{n+1}$ $\bullet^{n+m}$

$f.a$

$\varepsilon.a$

run ρ' of *Clock*

run ρ_1 of *Plant*

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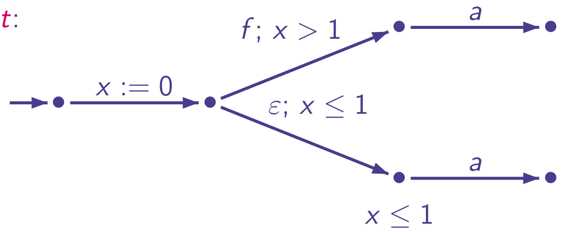

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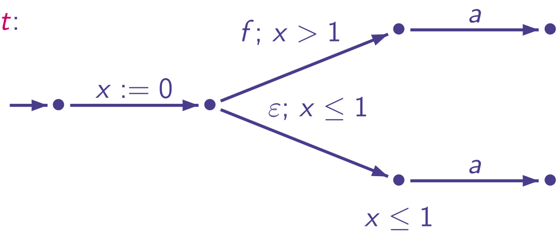

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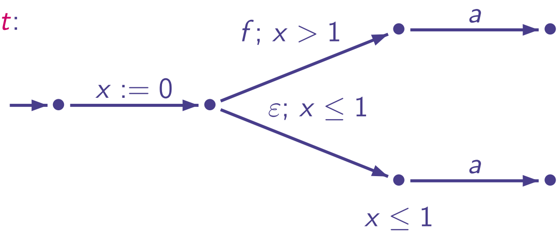

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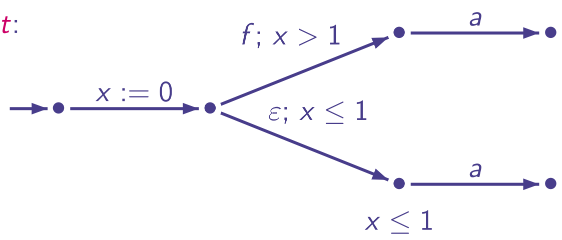

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$\rho_1 = t'.f.0.a.(2 - t')$ $\quad \rho_2 = 1.\varepsilon.0.a.1$

$\rho_1 \parallel \rho' = tick^n.f.a.tick^m$ $\quad \rho_2 \parallel \rho' \ni tick^n.\varepsilon.a.tick^m$

$tick^n.a.tick^m \in \text{Untimed}(\rho_1 \parallel \rho') \cap \text{Untimed}(\rho_2 \parallel \rho')$

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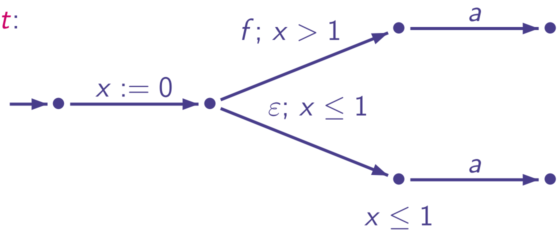
Plant:



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Outline

- Models for Timed Systems & Digital Clocks
- Monitoring with Digital Clocks
- Diagnosis with Digital Clocks
- **Conclusion & Open Problem**

Conclusion & Open Problem

- Monitoring with digital clocks: region graph
- (Δ, Clock) and *Clock*-diagnosability decidable
- Diagnosability (existence of a digital clock): Open
- Recent Related Work: [Jiang, Kumar, ACC'06]
  - Digital Clocks and Fault-Diagnosis
  - Periodic clock: ticks every $\Delta \pm \epsilon$
  - Problem 4 not considered

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Acknowledgements: ACI-CORTOS

Timed Automata [Alur & Dill, TCS'94]

A Timed Automaton $\mathcal{A}$ is a tuple $(L, \ell_0, \mathrm{Act}, X, \mathrm{inv}, \longrightarrow)$ where:

- L is a finite set of locations
- ℓ_0 is the initial location
- X is a finite set of clocks
- Act is a finite set of actions
- $\longrightarrow$ is a set of transitions of the form $\ell \xrightarrow{g\,,\,a\,,\,R} \ell'$ with:
  - $\ell, \ell' \in L$,
  - $a \in \mathrm{Act}$
  - a guard g which is a clock constraint over X
  - a reset set R which is the set of clocks to be reset to 0

Clock constraints are boolean combinations of $x \sim k$ with $x \in C$ and $k \in \mathbb{Z}$ and $\sim \in \{\leq, <\}$.

Back

Semantics of Timed Automata

Let $\mathcal{A} = (L, \ell_0, \mathrm{Act}, X, \mathrm{inv}, \longrightarrow)$ be a Timed Automaton.

A state (ℓ, v) of $\mathcal{A}$ is in $L \times \mathbb{R}_{\geq 0}^X$

The semantics of $\mathcal{A}$ is a Timed Transition System $S_{\mathcal{A}} = (Q, q_0, \mathrm{Act} \cup \mathbb{R}_{\geq 0}, \longrightarrow)$ with:

- $Q = L \times \mathbb{R}_{\geq 0}^X$
- $q_0 = (\ell_0, \overline{0})$
- $\longrightarrow$ consists in:

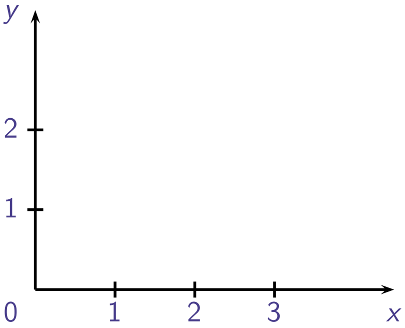
discrete transition: $(\ell, v) \xrightarrow{a} (\ell', v') \iff \begin{cases} \exists\, \ell \xrightarrow{g\,,\,a\,,\,r} \ell' \in \mathcal{A} \\ v \models g \\ v' = v[r \leftarrow 0] \\ v' \models \mathrm{inv}(\ell') \end{cases}$

delay transition: $(\ell, v) \xrightarrow{d} (\ell, v + d) \iff d \in \mathbb{R}_{\geq 0} \wedge v + d \models \mathrm{inv}(\ell)$

Back

The Region Abstraction

[Alur & Dill, TCS'94]



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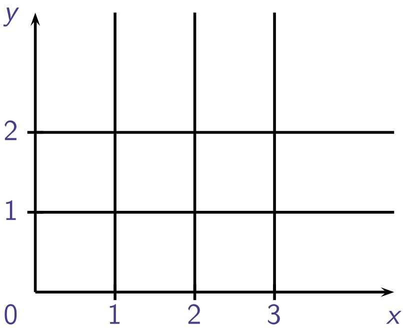

Build an equivalence relation which is of finite index and is:

- "compatible" with clock constraints ($g ::= x \sim c \quad g \wedge g$)

  $r, r' \in R \implies \forall$ constraints $g, \quad r \models g \iff r' \models g$

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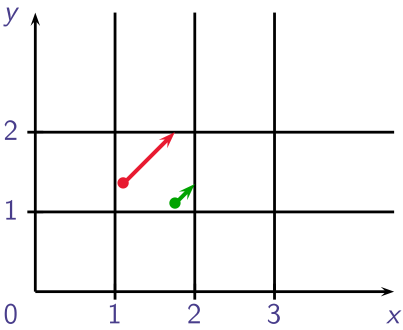

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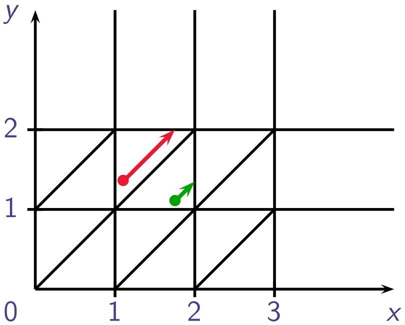

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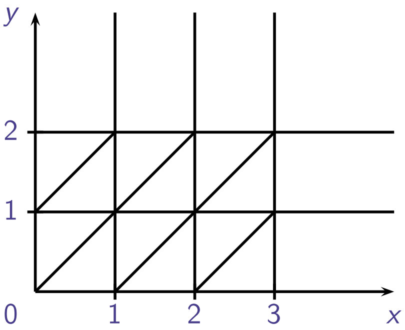

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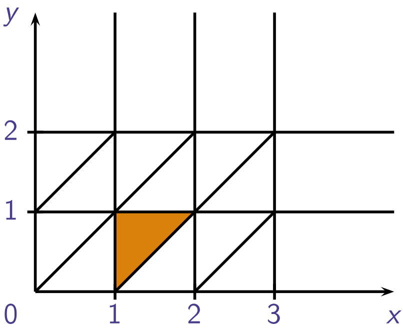

region defined by

$I_x =]1;2[\; I_y =]0;1[$

$\{x\} < \{y\}$

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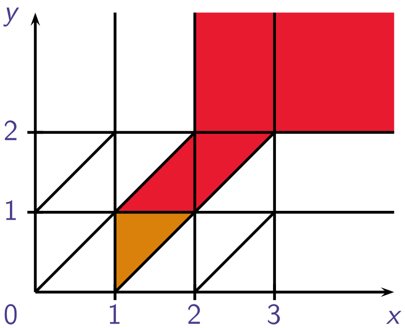

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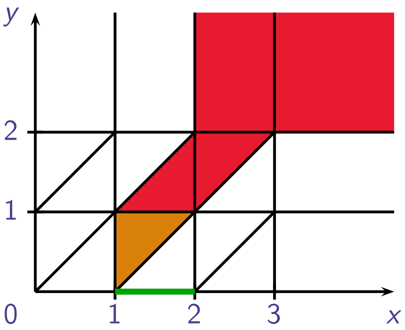

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delay successors

successor by reset

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The Region Automaton

- For each transition $\ell \xrightarrow{g,a,C:=0} \ell'$ of the TA
- Build transitions in the region automaton RA: $(\ell, R) \xrightarrow{a} (\ell', R')$ if:
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a TA and its region automaton RA are time-abstract bisimilar

- The region automaton is finite
- Language accepted by the RA = untimed language accepted by the TA
  a timed word $w = (a, 1.2)(b, 3.4)(a, 6.256)$; $\text{untimed}(w) = aba$
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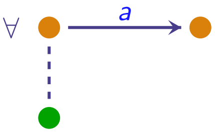
Time-abstract bisimulation



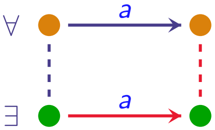
$(\ell_0, v_0) \xrightarrow{a_1, t_1} (\ell_1, v_1) \xrightarrow{a_2, t_2} (\ell_2, v_2) \xrightarrow{a_3, t_3} \cdots$

$(\ell_0, R_0) \xrightarrow{a_1} (\ell_1, R_1) \xrightarrow{a_2} (\ell_2, R_2) \xrightarrow{a_3} \cdots$

with $v_i \in R_i$ for all i.

Timed Auto.

Time-abstract bisimulation



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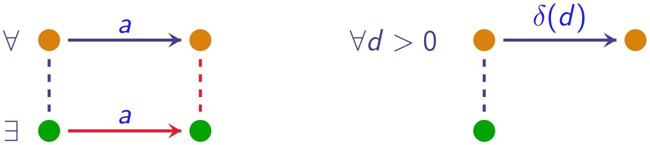

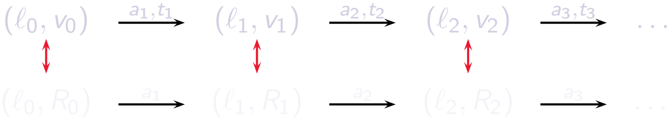

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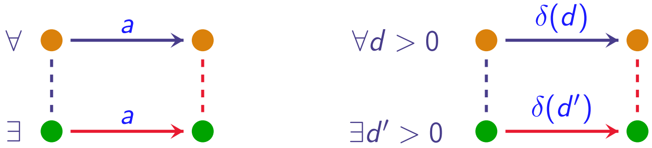

$$
\begin{array}{ccccccc}
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\updownarrow & & \updownarrow & & \updownarrow & & \\
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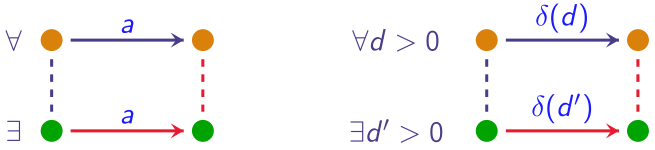

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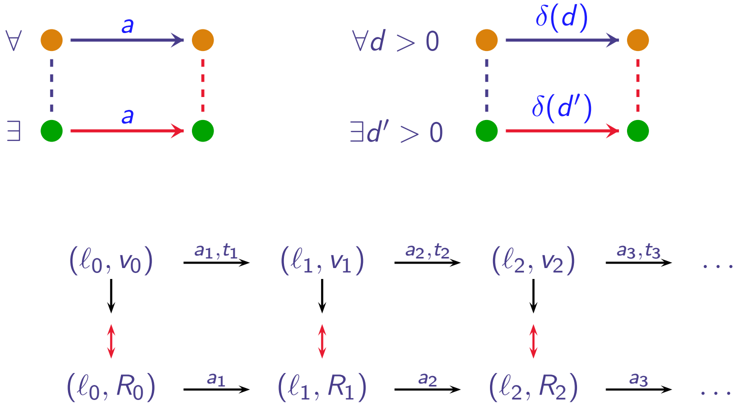
Time-abstract bisimulation



with $v_i \in R_i$ for all i.

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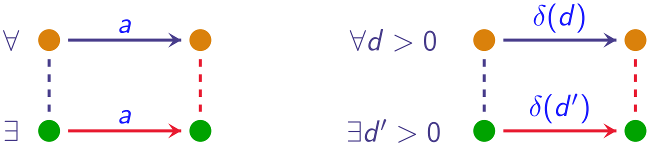

$$
\begin{array}{ccccccc}
(\ell_0, v_0) & \xrightarrow{a_1, t_1} & (\ell_1, v_1) & \xrightarrow{a_2, t_2} & (\ell_2, v_2) & \xrightarrow{a_3, t_3} & \dots \\
\updownarrow & & \updownarrow & & \updownarrow & & \\
(\ell_0, R_0) & \xrightarrow{a_1} & (\ell_1, R_1) & \xrightarrow{a_2} & (\ell_2, R_2) & \xrightarrow{a_3} & \dots
\end{array}
$$

with $v_i \in R_i$ for all i.

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Algorithm for Checking Diagnosability

Necessary and Sufficient Condition for Diagnosability:

$$
\begin{aligned}
A \text{ is not } \Sigma\text{-diagnosable} &\iff \forall k \in \mathbb{N}^*, A \text{ is not } (\Sigma, k)\text{-diagnosable} \\
&\iff \forall k \in \mathbb{N}^*, \begin{cases} \exists \rho \in \mathsf{NonFaulty}(A) \\ \exists \rho' \in \mathsf{Faulty}_{\geq k}(A) \\ \qquad \pi_{/\Sigma}(\rho) = \pi_{/\Sigma}(\rho') \end{cases}
\end{aligned}
$$

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Let $A_1 = (Q \times \{0,1\}, (q_0, 0), \Sigma^\varepsilon, \rightarrow_1)$ s.t.

- $(q,k) \xrightarrow{l}_1 (q',k')$ iff $q \xrightarrow{l} q'$ and $l \in \Sigma$ and $k = k'$;
- $(q,k) \xrightarrow{\varepsilon}_1 (q',1)$ iff $q \xrightarrow{f} q'$, (k is set to 1 after a fault occurs and will remain 1 once it has been set to 1);
- $(q,k) \xrightarrow{\varepsilon}_1 (q',k)$ iff $q \xrightarrow{\varepsilon} q'$.

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Define $A_2 = (Q, q_0, \Sigma^\varepsilon, \rightarrow_2)$ with

- $q \xrightarrow{l}_2 q'$ if $q \xrightarrow{l} q'$ and $l \in \Sigma$;
- $q \xrightarrow{\varepsilon}_2 q'$ if $q \xrightarrow{\varepsilon} q'$.

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$$\iff \forall k \in \mathbb{N}^*, \begin{cases} \exists \rho \in \text{NonFaulty}(A) \\ \exists \rho' \in \text{Faulty}_{\geq k}(A) \\ \qquad \pi_{/\Sigma}(\rho) = \pi_{/\Sigma}(\rho') \end{cases}$$

Let $\mathcal{B} = A_1 \times A_2$

Büchi acceptance condition: infinitely many faulty states and A_1-actions

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Theorem

$\mathrm{Lang}^\omega(\mathcal{B}) \neq \emptyset \iff A$ *is not* Σ*-diagnosable.*

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Theorem

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Theorem

The minimum k s.t. A is (Σ, k)-diagnosable can be computed in PTIME.