Efficient On-the-Fly Algorithms for Partially Observable Timed Games

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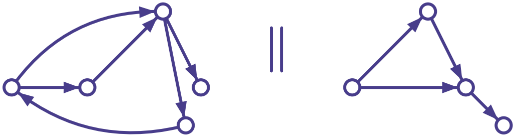
and Analysis of Timed Systems
FORMATS'07

Salzburg, Austria

Verification and Control

Verification and Control

Modelling



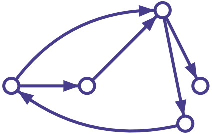
$$S \parallel C \models \varphi$$

Always (not bad)

Verification and Control

Does the system meet the specification?

Modelling



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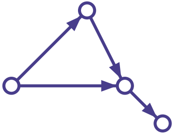

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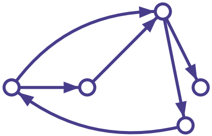

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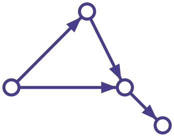


Always (not bad)

$S \quad || \quad C \quad \models \quad \varphi$

Model Checking Problem

Does the closed system $(S \mid\mid C)$ satisfy φ ?

Verification and Control

Can we enforce the system to meet the specification?

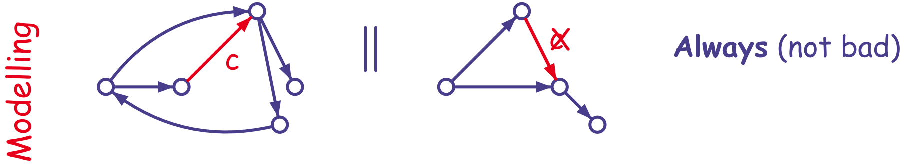
Modelling

Always (not bad)

$$S \parallel C \models \varphi$$

Verification and Control

Can we enforce the system to meet the specification?



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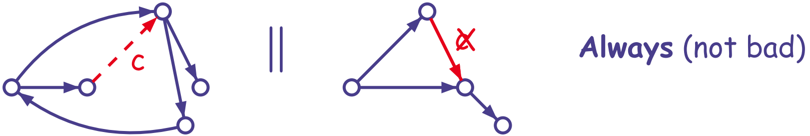


$S \quad \| \quad C \quad \models \quad \varphi$

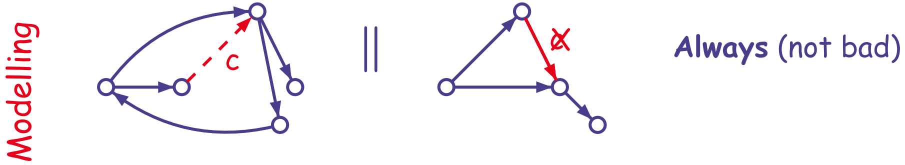
Control Problem

Can the open system S be restricted to satisfy φ ?

Is there a Controller C such that $(S \parallel C) \models \varphi$?

Verification and Control

Can we enforce the system to meet the specification?



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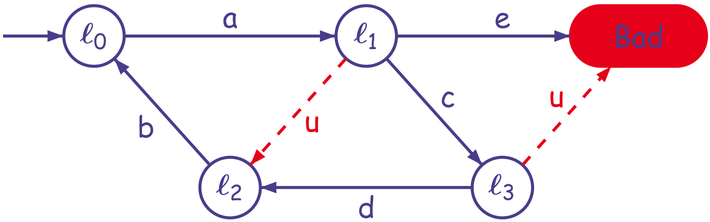
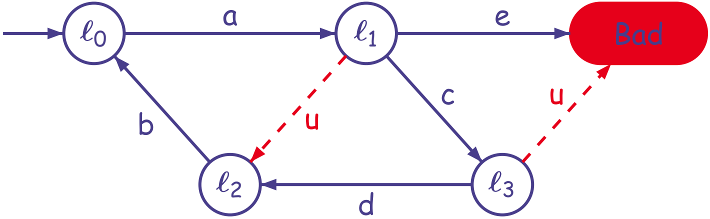
Control Problems as Games



- Control problem = game = controller (C) vs environment (E)
- Various types of game models for C and E
  - Finite or pushdown or counter automata ...
  - Timed (or hybrid) automata
- Goal: find a strategy for the controller to win
  Avoid bad states: safety control
  Enforce good states: reachability control

Control Problems as Games



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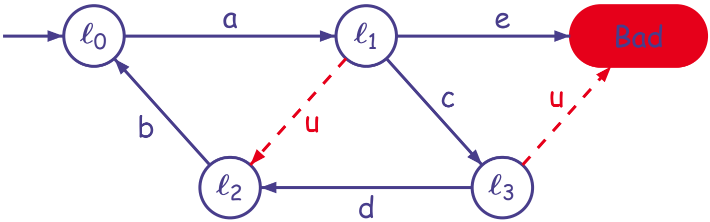

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Outline of the Talk

- Reachability Control
  - Finite-State Games
  - Backward Algorithm for Finite State Games
  - Backward Algorithm for Timed Games
  - Summary of the Results for (Reachability) Control
- On-the-fly Algorithms for Reachability Control
  - Finite State Games
  - Timed Games
- Implementation, Optimizations, Time Optimality
- Partial Observation
  - Results About Partial Observation for Timed Games
  - Stuttering Free Observations
  - Knowledge Based Subset Construction

Next:

- Reachability Control
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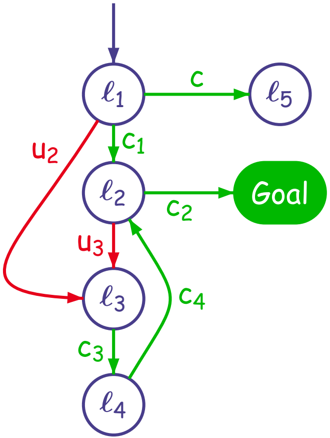
Reachability Control for Finite Games

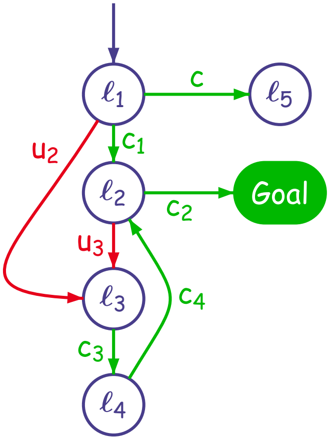


$\rightarrow$ Uncontrollable
$\rightarrow$ Controllable

Aim: enforce Goal

- Semantics: no priority
  Cont. must take a controllable action
- Winning run = a run containing Goal
- Strategy: based on the full history tells which controllable action to fire
  It restricts the set of behaviors of the open system
- Winning strategy: all the runs in the controlled system are winning
- Winning state = a state from which there is winning strategy

Reachability Control for Finite Games



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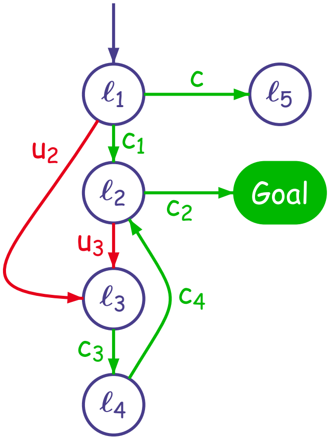

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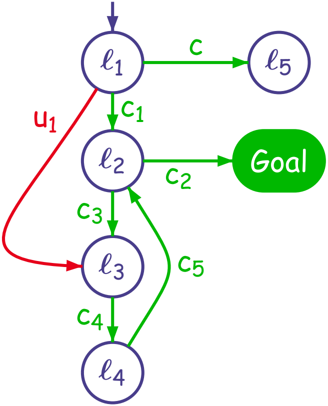
How to Solve Reachability Games?

Backward Computation of Winning States



$\overline{X}$ = complement of X

Controllable Predecessors:

$$\pi(X) = \big(cPred(X) \setminus uPred(\overline{X})\big)$$

Iterate π: $X_{i+1} = X_i \cup \pi(X_i)$

1. $X_0 = \{Goal\}$
2. $X_1 = \{Goal, \ell_2\}$
3. $X_2 = \{Goal, \ell_2, \ell_4\}$
4. $X_3 = \{Goal, \ell_2, \ell_4, \ell_3\}$
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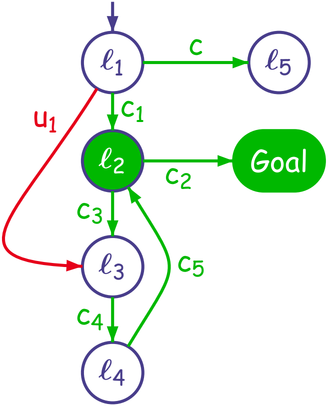

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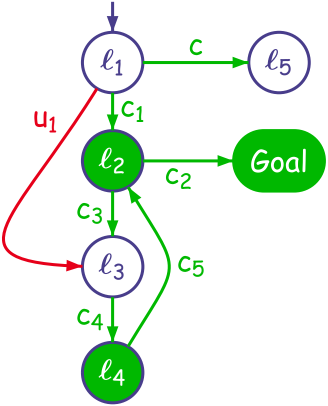

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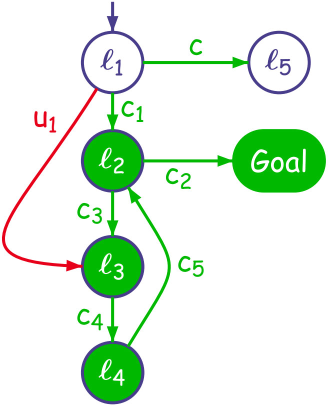

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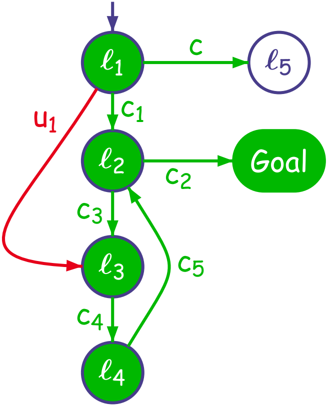

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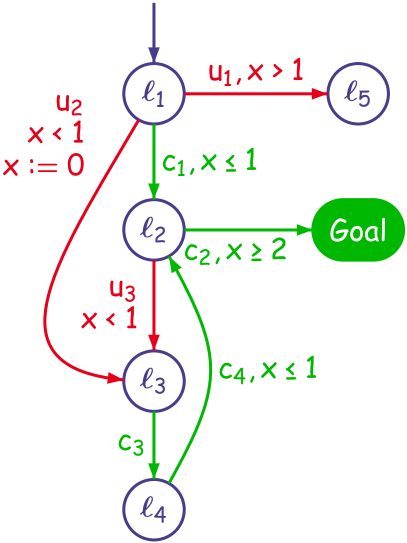
Reachability Control for Timed Games



Safe Time Elapsing:
When is it safe to let time elapse from q to q′ ?

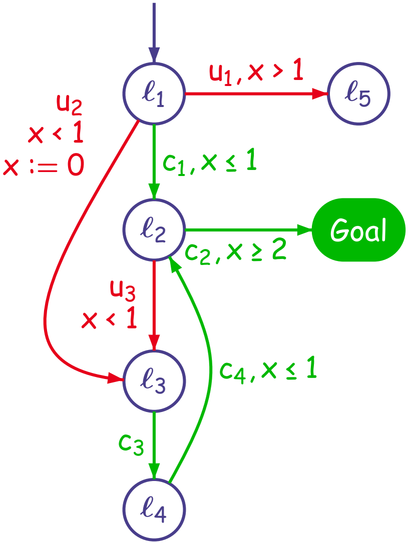
q $\qquad$ $q' \in X$

Controllable Predecessors:

$$\pi(X) = \mathrm{Pred}_t\big(X \cup \mathrm{cPred}(X), \mathrm{uPred}(\overline{X})\big)$$

▸ Timed Auto

Reachability Control for Timed Games



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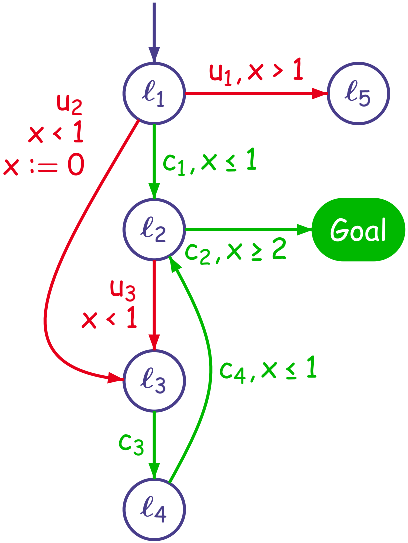

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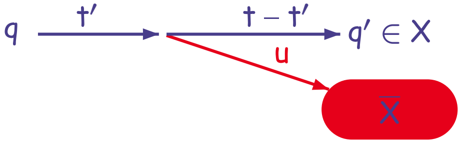

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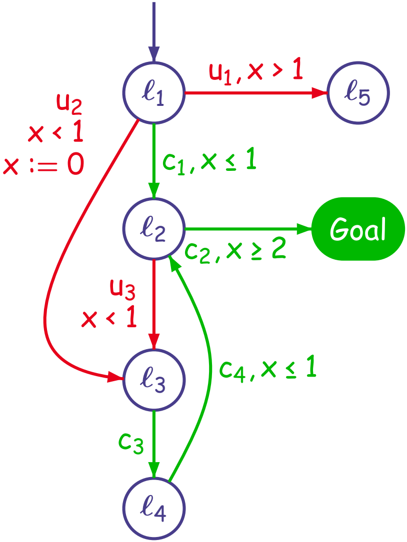

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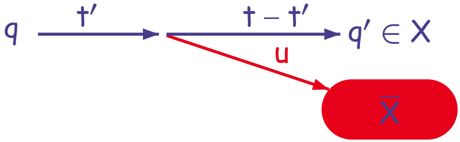

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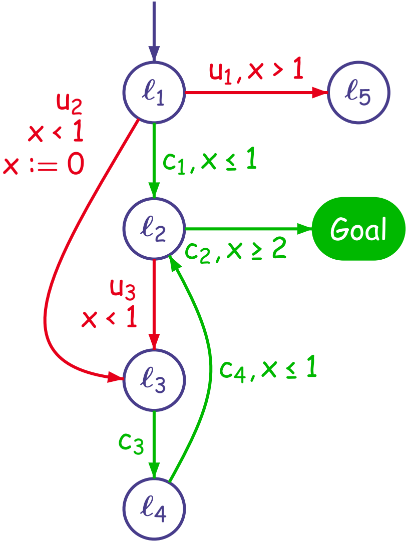

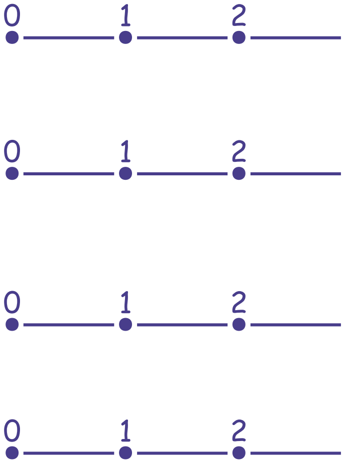

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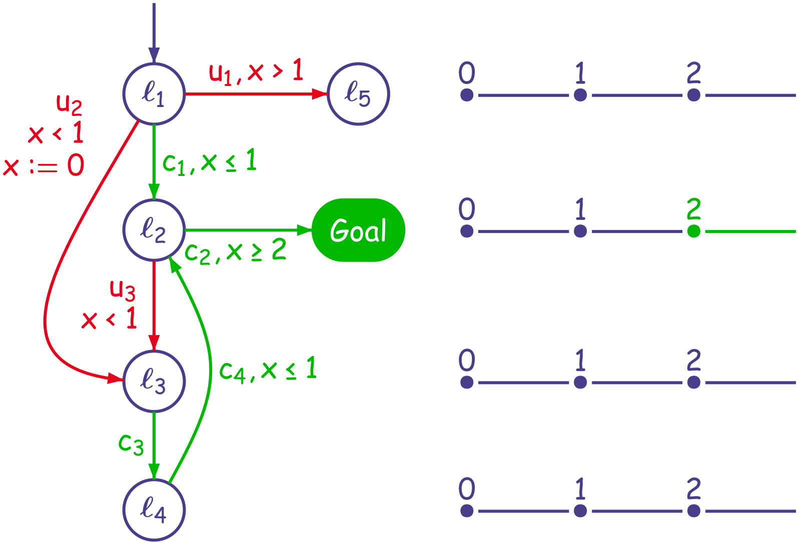

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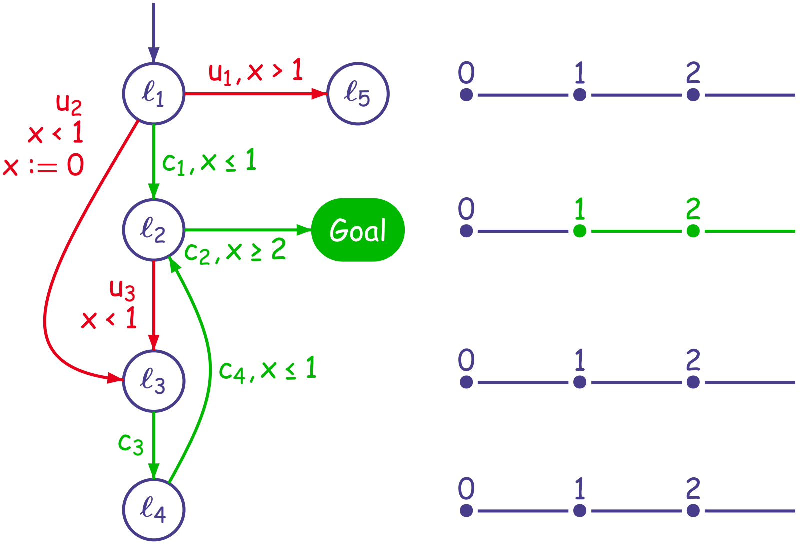

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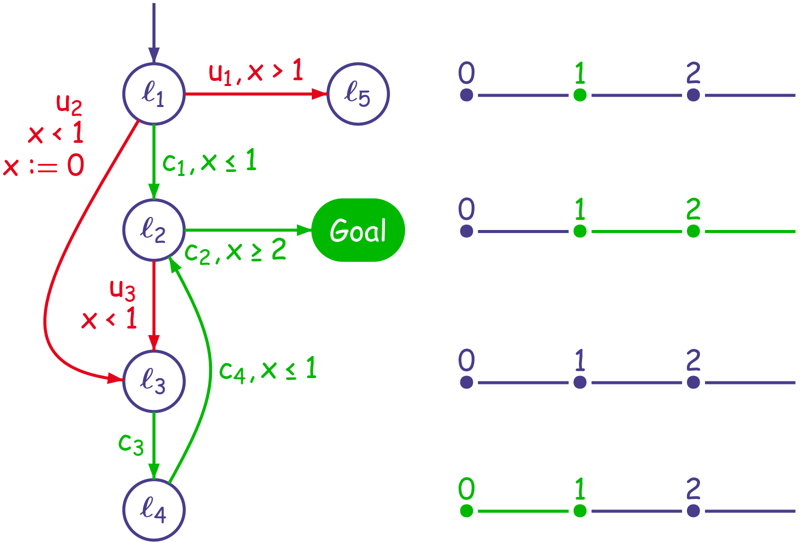

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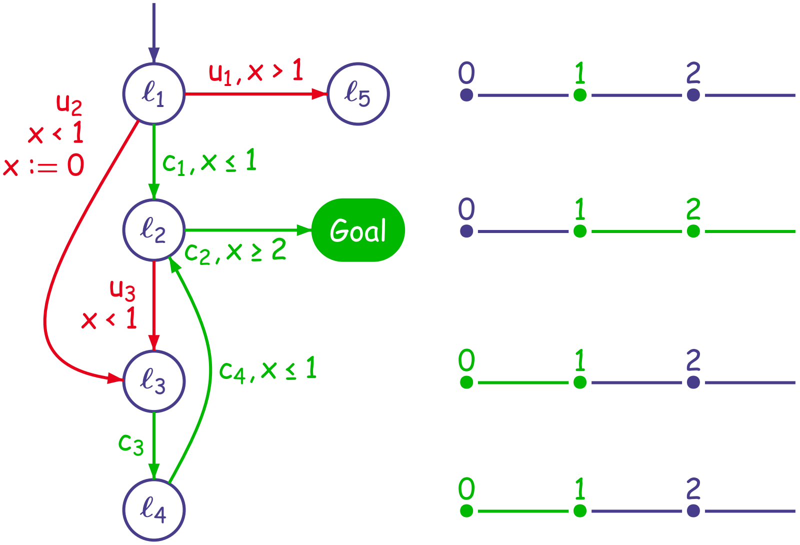

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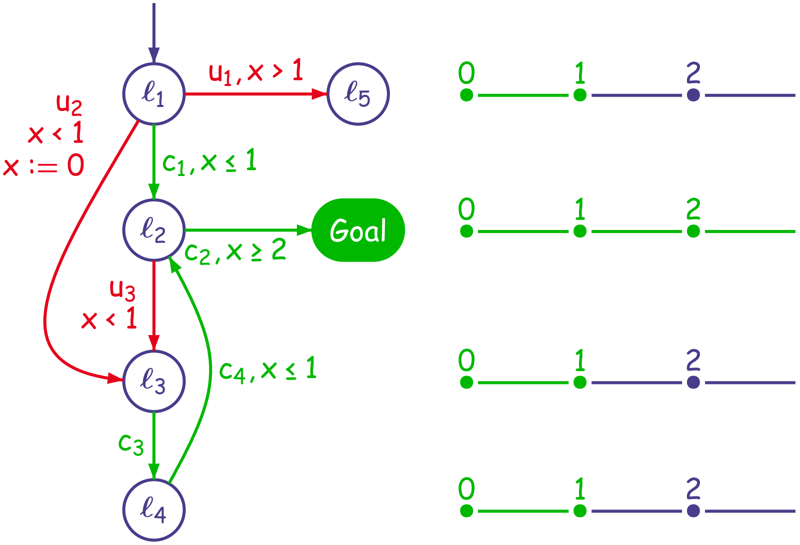

State-of-the-art

Known Results for Timed (Game) Automata:

- Reachability in Timed Automata [Alur & Dill'94]
- Büchi Control for Timed Game Automata [Maler et al.'95]
- Time Optimal Control [Asarin & Maler'99]
- Optimal Control for Priced Timed Game Automata [Bouyer et al.'04]
- Half on-the-fly algorithm [Altisen & Tripakis'99, Altisen & Tripakis'02]

Our Contribution: True On-the-fly algorithm for reachability games

- Advantages: [Concur'05]
  - avoid constructing all backward & forward reachable states
  - allows for use of discrete variables (e.g. $i := i + 1$)
- Extends to Time-Optimal Control
- Extends to Partially Observable Games [ATVA'07]
- Efficient implementation in UPPAAL-TiGA [UPPAAL-TiGA'07]

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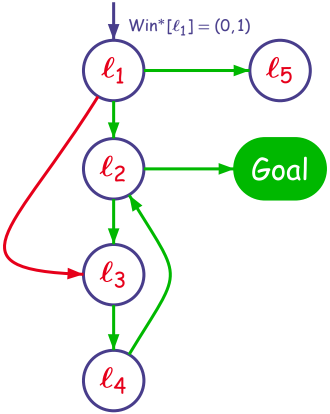
Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

Initialization:
  Passed $\leftarrow \{q_0\}$;
  Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
  Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
  Depend$[q_0] \leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
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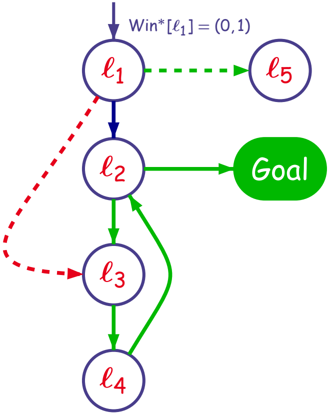

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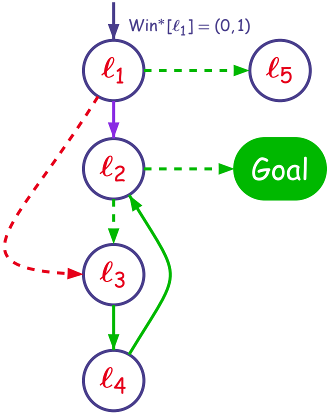

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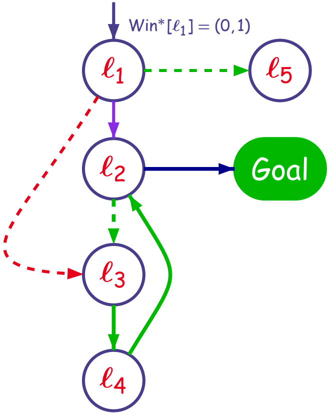
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_2, c_2, Goal)$$

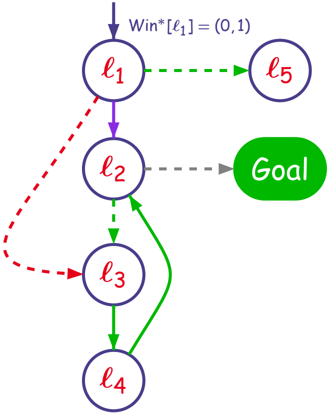
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

```
while ((Waiting ≠ ∅) ∧ Win[q0] ≠ 1)) do
    e = (q, a, q') ← pop(Waiting);
    if q' ∉ Passed then {
        Passed ← Passed ∪ {q'};
        Depend[q'] ← {(q, a, q')};
        Win[q'] ← (q' ∈ Goal ? 1 : 0);
        Waiting ← Waiting ∪ {(q', a, q'') | q' -a-> q''};
        Win*[q] ← (0, #{q -u->});
        if Win[q'] then Waiting ← Waiting ∪ {e};
    }
    else (* reevaluate *)
        Win*[q] ← Update(Win*[q]) ;
        if (Win*[q] = (k, 0) ∧ k ≥ 1) then {
            Waiting ← Waiting ∪ Depend[q];
            Win[q] ← 1;
        }
        if Win[q'] = 0 then Depend[q'] ← Depend[q'] ∪ {e};
    endif
endwhile
```

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current

$\longrightarrow$ Explored Forwards

Dashed = in Waiting List

$$e = (\ell_2, c_2, Goal)$$

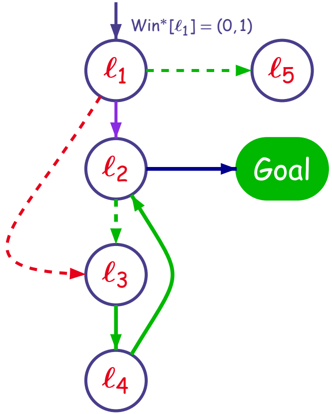
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current

$\longrightarrow$ Explored Forwards

Dashed = in Waiting List

$$e = (\ell_2, c_2, Goal)$$

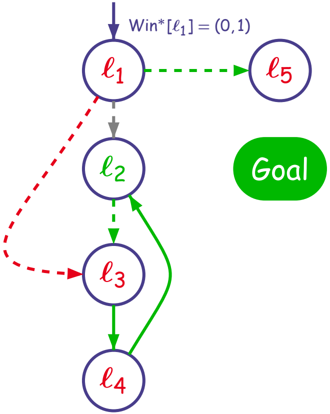
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, a, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, a, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current

$\longrightarrow$ Explored Forwards

Dashed = in Waiting List

$$e = (\ell_2, c_2, Goal)$$

Initialization:

$\quad$ Passed $\leftarrow \{q_0\}$;

$\quad$ Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;

$\quad$ Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;

$\quad$ Depend$[q_0] \leftarrow \emptyset$;

Main:

while $((\text{Waiting} \neq \emptyset) \wedge \text{Win}[q_0] \neq 1))$ **do**

$\quad e = (q, \alpha, q') \leftarrow$ pop(Waiting);

$\quad$ **if** $q' \notin$ Passed **then** {

$\qquad$ Passed $\leftarrow$ Passed $\cup \{q'\}$;

$\qquad$ Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;

$\qquad$ Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;

$\qquad$ Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;

$\qquad$ Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;

$\qquad$ **if** Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;

$\quad$ }

$\quad$ **else (* reevaluate *)**

$\qquad$ Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;

$\qquad$ **if** (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {

$\qquad\quad$ Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;

$\qquad\quad$ Win$[q] \leftarrow 1$;

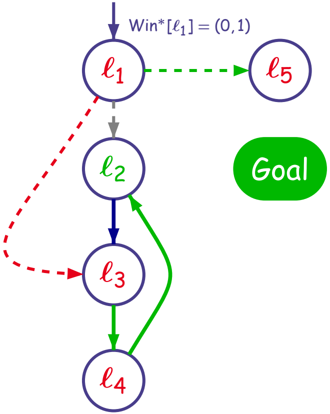
$\qquad$ }

$\qquad$ **if** Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;

$\quad$ **endif**

endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



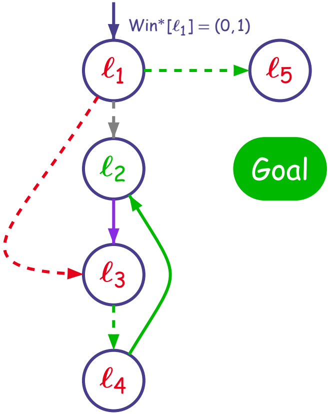
→ Current
→ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_2, c_3, \ell_3)$$

Initialization:
Passed ← $\{q_0\}$;
Waiting ← $\{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
$Win[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
$Depend[q_0] \leftarrow \emptyset$;

Main:
while $((Waiting \neq \emptyset) \wedge Win[q_0] \neq 1)$ **do**
  $e = (q, a, q') \leftarrow pop(Waiting)$;
  if $q' \notin Passed$ **then {**
    $Passed \leftarrow Passed \cup \{q'\}$;
    $Depend[q'] \leftarrow \{(q, a, q')\}$;
    $Win[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    $Waiting \leftarrow Waiting \cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
    $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if $Win[q']$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
  }
  else (* reevaluate *)
    $Win^*[q] \leftarrow$ **Update**$(Win^*[q])$;
    if $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then {**
      $Waiting \leftarrow Waiting \cup Depend[q]$;
      $Win[q] \leftarrow 1$;
    }
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



→ Current
→ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_2, c_3, \ell_3)$$

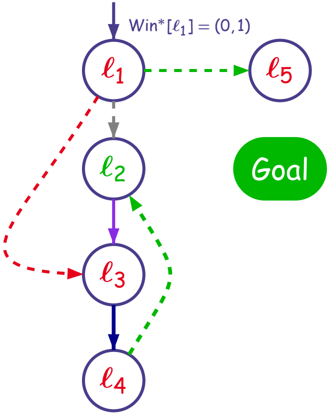
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then {**
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win*$[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win*$[q] \leftarrow$ **Update**(Win*$[q]$) ;
    if (Win*$[q] = (k, 0) \wedge k \geq 1$) **then {**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



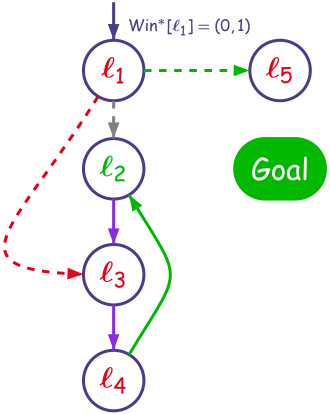
$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_2, c_3, \ell_3)$$

Initialization:
Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win[q_0] $\leftarrow$ ($q_0 \in$ Goal ? 1 : 0);
Depend[q_0] $\leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ Win[q_0] $\neq$ 1)) **do**
  $e = (q, a, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend[q'] $\leftarrow \{(q, a, q')\}$;
    Win[q'] $\leftarrow$ ($q' \in$ Goal ? 1 : 0);
    Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
    Win*[q] $\leftarrow$ (0, #{$q \xrightarrow{u}$});
    if Win[q'] **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* **reevaluate** *)
    Win*[q] $\leftarrow$ **Update**(Win*[q]) ;
    if (Win*[q] = $(k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend[q];
      Win[q] $\leftarrow$ 1;
    }
    if Win[q'] = 0 **then** Depend[q'] $\leftarrow$ Depend[q'] $\cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_2, c_3, \ell_3)$$

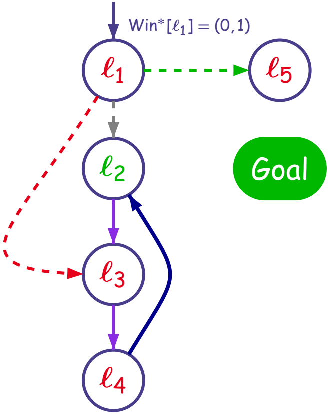
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge Win[q_0] \neq 1))$ **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



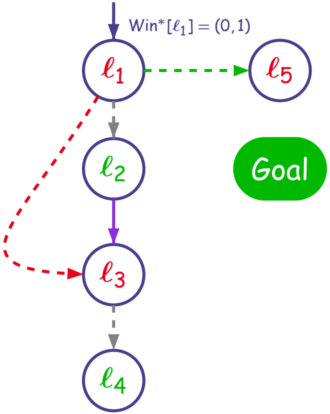
$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_4, c_5, \ell_2)$$

Initialization:
Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset) \wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



→ Current
→ Explored Forwards
Dashed = in Waiting List

$e = (\ell_4, c_5, \ell_2)$

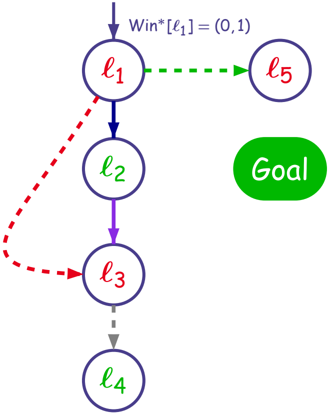
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, a, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, a, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
    Win*$[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win*$[q] \leftarrow$ **Update**(Win*$[q]$) ;
    if (Win*$[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current

$\longrightarrow$ Explored Forwards

Dashed = in Waiting List

$$e = (\ell_1, c_1, \ell_2)$$

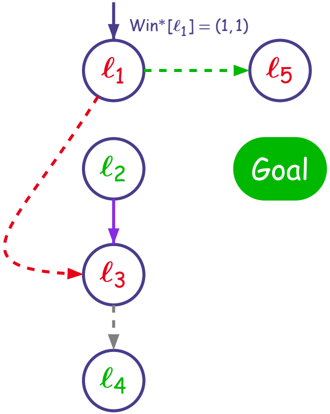
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
$Win[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
$Depend[q_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge Win[q_0] \neq 1))$ **do**
$e = (q, a, q') \leftarrow pop(Waiting)$;
if $q' \notin Passed$ **then** {
Passed $\leftarrow$ Passed $\cup \{q'\}$;
$Depend[q'] \leftarrow \{(q, a, q')\}$;
$Win[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
$Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
if $Win[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
}
else (* reevaluate *)
$Win^*[q] \leftarrow$ **Update**$(Win^*[q])$;
if $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then** {
Waiting $\leftarrow$ Waiting $\cup$ Depend[q];
$Win[q] \leftarrow 1$;
}
if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_1, c_1, \ell_2)$$

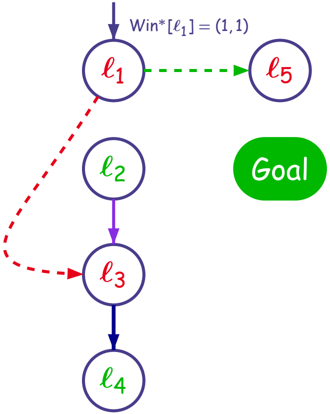
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
 $e = (q, a, q') \leftarrow$ pop(Waiting);
 if $q' \notin$ Passed **then** {
  Passed $\leftarrow$ Passed $\cup \{q'\}$;
  Depend$[q'] \leftarrow \{(q, a, q')\}$;
  Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
  Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
  Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
  if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
 }
 else (* reevaluate *)
  Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
  if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
   Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
   Win$[q] \leftarrow 1$;
  }
  if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
 endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

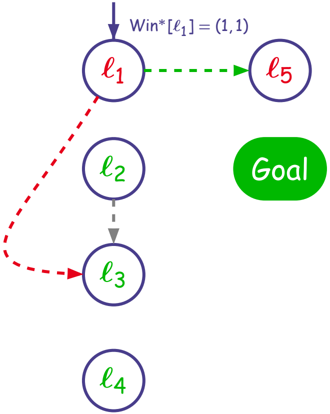
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

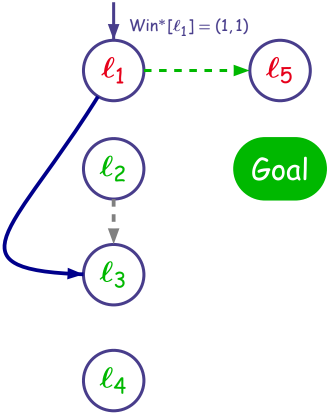
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, \alpha, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    Win*$[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win*$[q] \leftarrow$ **Update**(Win*$[q]$) ;
    if (Win*$[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

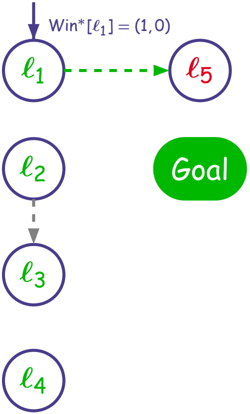
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
$Depend[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ $Win[q_0] \neq 1$)) **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \not\in$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    $Depend[q'] \leftarrow \{(q, \alpha, q')\}$;
    $Win[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if $Win[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    $Win^*[q] \leftarrow$ **Update**$(Win^*[q])$;
    if $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then** {
      Waiting $\leftarrow$ Waiting $\cup$ $Depend[q]$;
      $Win[q] \leftarrow 1$;
    }
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



$\longrightarrow$ Current

$\longrightarrow$ Explored Forwards

Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

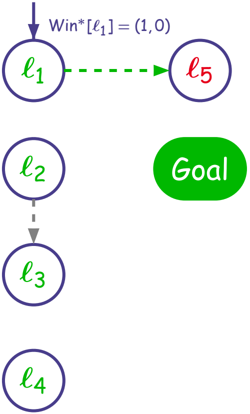
Initialization:

Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
  $e = (q, a, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    Depend$[q'] \leftarrow \{(q, a, q')\}$;
    Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
    Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
    if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
      Win$[q] \leftarrow 1$;
    }
    if Win$[q'] = 0$ **then** Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu & Smolka'98]



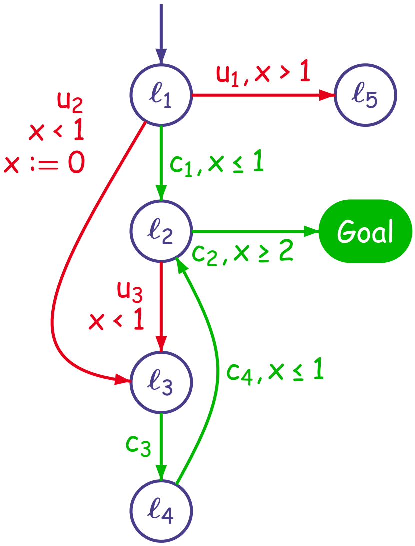
$\longrightarrow$ Current
$\longrightarrow$ Explored Forwards
Dashed = in Waiting List

$$e = (\ell_3, c_4, \ell_4)$$

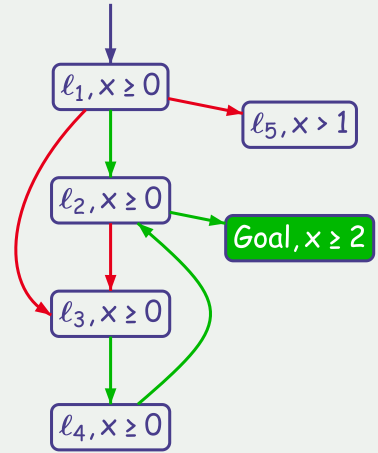
Initialization:
Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
$Depend[q_0] \leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ $Win[q_0] \neq 1$)) **do**
  $e = (q, \alpha, q') \leftarrow$ pop(Waiting);
  if $q' \notin$ Passed **then** {
    Passed $\leftarrow$ Passed $\cup \{q'\}$;
    $Depend[q'] \leftarrow \{(q, \alpha, q')\}$;
    $Win[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
    Waiting $\leftarrow$ Waiting $\cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if $Win[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  }
  else (* reevaluate *)
    $Win^*[q] \leftarrow$ **Update**($Win^*[q]$) ;
    if ($Win^*[q] = (k, 0) \wedge k \geq 1$) **then** {
      Waiting $\leftarrow$ Waiting $\cup$ $Depend[q]$;
      $Win[q] \leftarrow 1$;
    }
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

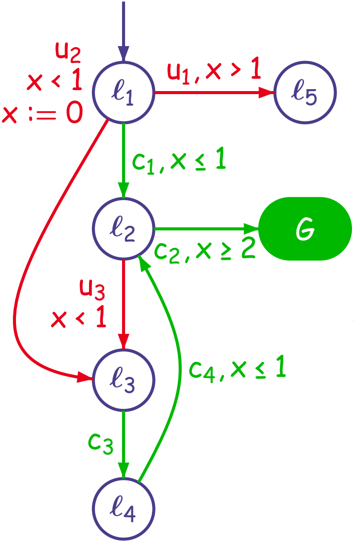
On-The-Fly algorithm for Timed Games (1)



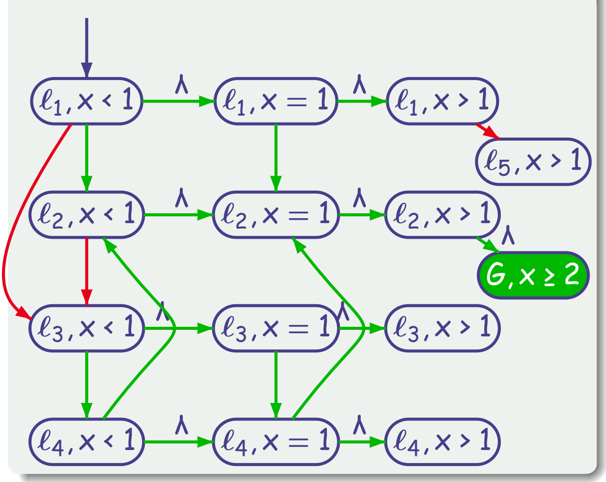
Using the Simulation Graph



Second Try (2) [Altisen & Tripakis'99, Altisen & Tripakis'02]



Stable Partitionning



Towards a True On-The-Fly Algorithm

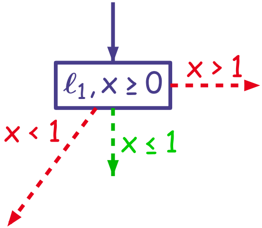
Our Approach:

- Write a Symbolic version of Liu & Smolka
- Use Symbolic states and Transitions
- Apply this to Timed Games

Key issues to be adressed:

- Symbolic States can be partially winning
  compared to finite state games where 0 or 1
- When to propagate backwards ?
- Termination, Complexity ?

Liu & Smolka for Timed Games



Initialization:

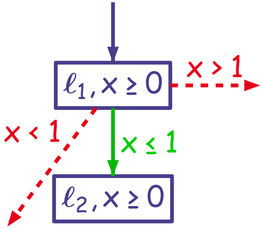
Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ (($\ell_0, 0) \notin$ Win$[S_0]$)) **do**
  $e = (S, a, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend$[S'] \leftarrow \{(S, a, S')\}$;
    Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else **(* reevaluate *)**
    Win$^* \leftarrow$ Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
      $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
    if (Win$[S] \subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

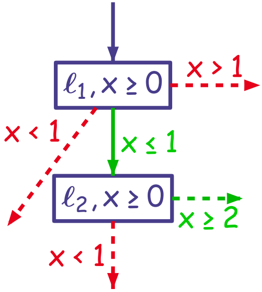
Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, \alpha, S') \mid S' = Post_\alpha(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ ((ℓ_0, 0) $\notin$ Win$[S_0]$)) **do**
 e = $(S, \alpha, S') \leftarrow$ pop(Waiting);
 if $S' \notin$ Passed **then**
  Passed $\leftarrow$ Passed $\cup \{S'\}$;
  Depend$[S'] \leftarrow \{(S, \alpha, S')\}$;
  Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
  Waiting $\leftarrow$ Waiting $\cup \{(S', \alpha, S'') \mid S'' = Post_\alpha(S')^{\nearrow}\}$;
  if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
 else (* reevaluate *)
  Win$^* \leftarrow$Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
   $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
  if (Win$[S] \subsetneq$ Win*) **then**
   Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
  Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
 endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

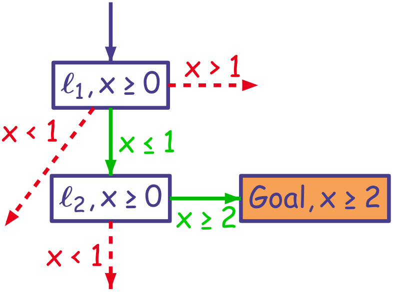
$\quad$ Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
$\quad$ Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
$\quad$ Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
$\quad$ Depend$[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ ((ℓ_0, 0) $\notin$ Win$[S_0]$)) **do**
$\quad$ $e = (S, a, S') \leftarrow$ pop(Waiting);
$\quad$ **if** $S' \notin$ Passed **then**
$\quad\quad$ Passed $\leftarrow$ Passed $\cup \{S'\}$;
$\quad\quad$ Depend$[S'] \leftarrow \{(S, a, S')\}$;
$\quad\quad$ Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
$\quad\quad$ Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
$\quad\quad$ **if** Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
$\quad$ **else (* reevaluate *)**
$\quad\quad$ Win$^* \leftarrow$ Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
$\quad\quad\quad \bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
$\quad\quad$ **if** (Win$[S] \subsetneq$ Win*) **then**
$\quad\quad\quad$ Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
$\quad\quad$ Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
$\quad$ **endif**
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

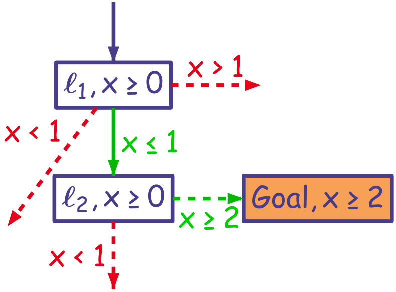
$Passed \leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
$Waiting \leftarrow \{(S_0, \alpha, S') \mid S' = Post_\alpha(S_0)^{\nearrow}\}$;
$Win[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
$Depend[S_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge ((\ell_0, 0) \notin Win[S_0]))$ **do**
  $e = (S, \alpha, S') \leftarrow pop(Waiting)$;
  if $S' \notin Passed$ **then**
    $Passed \leftarrow Passed \cup \{S'\}$;
    $Depend[S'] \leftarrow \{(S, \alpha, S')\}$;
    $Win[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    $Waiting \leftarrow Waiting \cup \{(S', \alpha, S'') \mid S'' = Post_\alpha(S')^{\nearrow}\}$;
    if $Win[S'] \neq \emptyset$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
  else (* reevaluate *)
    $Win^* \leftarrow Pred_t(Win[S] \cup \bigcup_{S \xrightarrow{c} T} cPred(Win[T]),$
      $\bigcup_{S \xrightarrow{u} T} uPred(T \setminus Win[T])) \cap S$;
    if $(Win[S] \subsetneq Win^*)$ **then**
      $Waiting \leftarrow Waiting \cup Depend[S]$; $Win[S] \leftarrow Win^*$;
    $Depend[S'] \leftarrow Depend[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

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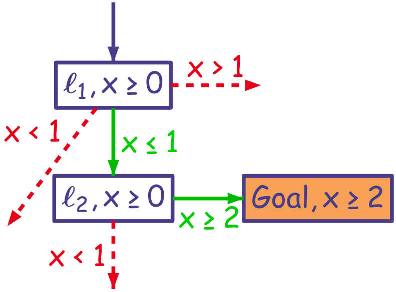
$\quad$ Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
$\quad$ Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
$\quad$ Depend$[S_0] \leftarrow \emptyset$;

Main:

while $((\text{Waiting} \neq \emptyset) \wedge ((\ell_0, 0) \not\in \text{Win}[S_0]))$ **do**
$\quad e = (S, a, S') \leftarrow$ pop(Waiting);
$\quad$ **if** $S' \not\in$ Passed **then**
$\quad\quad$ Passed $\leftarrow$ Passed $\cup \{S'\}$;
$\quad\quad$ Depend$[S'] \leftarrow \{(S, a, S')\}$;
$\quad\quad$ Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
$\quad\quad$ Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
$\quad\quad$ **if** Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
$\quad$ **else (* reevaluate *)**
$\quad\quad$ $\text{Win}^* \leftarrow \text{Pred}_t(\text{Win}[S] \cup \bigcup_{S \xrightarrow{c} T} \text{cPred}(\text{Win}[T]), \bigcup_{S \xrightarrow{u} T} \text{uPred}(T \setminus \text{Win}[T])) \cap S$;
$\quad\quad$ **if** $(\text{Win}[S] \subsetneq \text{Win}^*)$ **then**
$\quad\quad\quad$ Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
$\quad\quad$ Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
$\quad$ **endif**
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

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Main:

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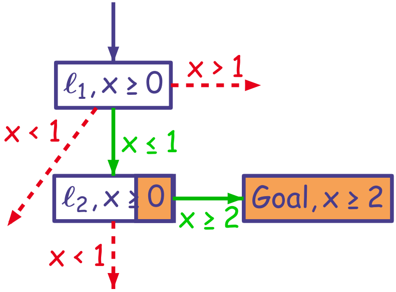
  $e = (S, a, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend$[S'] \leftarrow \{(S, a, S')\}$;
    Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    Win$^* \leftarrow$Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
      $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
    if (Win$[S] \subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



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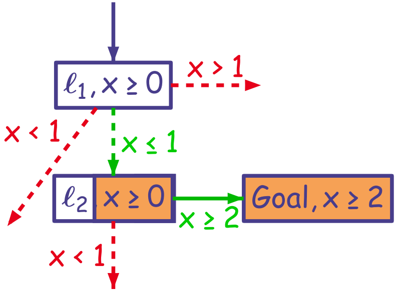
while ((Waiting $\neq \emptyset$) $\wedge$ (($\ell_0, 0$) $\notin$ Win$[S_0]$)) **do**
  $e = (S, a, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend$[S'] \leftarrow \{(S, a, S')\}$;
    Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    Win$^* \leftarrow$ Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$), $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
    if (Win$[S] \subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

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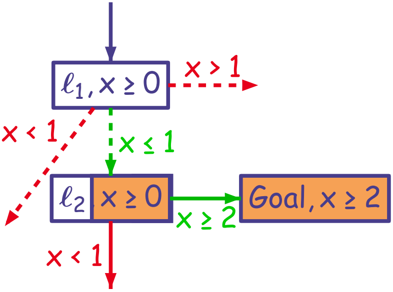
Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ ((ℓ_0, 0) $\notin$ Win$[S_0]$)) **do**
  $e = (S, a, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend$[S'] \leftarrow \{(S, a, S')\}$;
    Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    Win$^* \leftarrow$ Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
      $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
    if (Win$[S] \subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
$\mathrm{Win}[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
$\mathrm{Depend}[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ ($(\ell_0, 0) \notin \mathrm{Win}[S_0]$)) **do**
 $e = (S, a, S') \leftarrow$ pop(Waiting);
 if $S' \notin$ Passed **then**
  Passed $\leftarrow$ Passed $\cup \{S'\}$;
  $\mathrm{Depend}[S'] \leftarrow \{(S, a, S')\}$;
  $\mathrm{Win}[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
  Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
  if $\mathrm{Win}[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
 else (* reevaluate *)
  $\mathrm{Win}^* \leftarrow \mathrm{Pred}_t(\mathrm{Win}[S] \cup \bigcup_{S \xrightarrow{c} T} \mathrm{cPred}(\mathrm{Win}[T]),$
   $\bigcup_{S \xrightarrow{u} T} \mathrm{uPred}(T \setminus \mathrm{Win}[T])) \cap S$;
  if $(\mathrm{Win}[S] \subsetneq \mathrm{Win}^*)$ **then**
   Waiting $\leftarrow$ Waiting $\cup$ Depend[S]; $\mathrm{Win}[S] \leftarrow \mathrm{Win}^*$;
  $\mathrm{Depend}[S'] \leftarrow \mathrm{Depend}[S'] \cup \{e\}$;
 endif
endwhile

Skip algorithm

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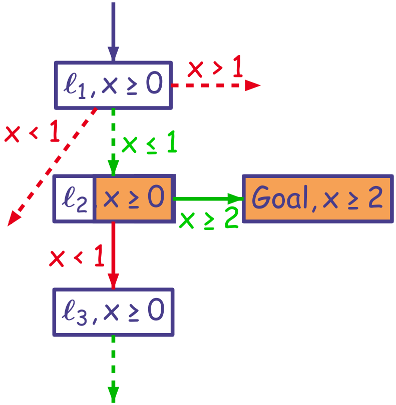

Initialization:

$\mathrm{Passed} \leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
$\mathrm{Waiting} \leftarrow \{(S_0, \alpha, S') \mid S' = \mathit{Post}_\alpha(S_0)^{\nearrow}\}$;
$\mathrm{Win}[S_0] \leftarrow S_0 \cap (\{\mathrm{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
$\mathrm{Depend}[S_0] \leftarrow \emptyset$;

Main:

while $((\mathrm{Waiting} \neq \emptyset) \wedge ((\ell_0, 0) \not\in \mathrm{Win}[S_0]))$ **do**
$e = (S, \alpha, S') \leftarrow \mathrm{pop}(\mathrm{Waiting})$;
if $S' \not\in \mathrm{Passed}$ **then**
$\mathrm{Passed} \leftarrow \mathrm{Passed} \cup \{S'\}$;
$\mathrm{Depend}[S'] \leftarrow \{(S, \alpha, S')\}$;
$\mathrm{Win}[S'] \leftarrow S' \cap (\{\mathrm{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
$\mathrm{Waiting} \leftarrow \mathrm{Waiting} \cup \{(S', \alpha, S'') \mid S'' = \mathit{Post}_\alpha(S')^{\nearrow}\}$;
if $\mathrm{Win}[S'] \neq \emptyset$ **then** $\mathrm{Waiting} \leftarrow \mathrm{Waiting} \cup \{e\}$;
else (* reevaluate *)
$\mathrm{Win}^* \leftarrow \mathrm{Pred}_t(\mathrm{Win}[S] \cup \bigcup_{S \xrightarrow{c} T} \mathrm{cPred}(\mathrm{Win}[T]), \bigcup_{S \xrightarrow{u} T} \mathrm{uPred}(T \setminus \mathrm{Win}[T])) \cap S$;
if $(\mathrm{Win}[S] \subsetneq \mathrm{Win}^*)$ **then**
$\mathrm{Waiting} \leftarrow \mathrm{Waiting} \cup \mathrm{Depend}[S]$; $\mathrm{Win}[S] \leftarrow \mathrm{Win}^*$;
$\mathrm{Depend}[S'] \leftarrow \mathrm{Depend}[S'] \cup \{e\}$;
endif
endwhile

Skip algorithm

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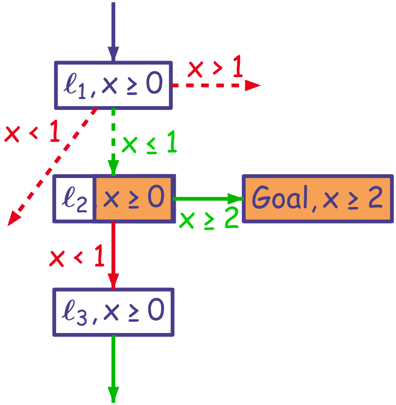

Initialization:

$\quad Passed \leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
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$\quad Win[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
$\quad Depend[S_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge ((\ell_0, 0) \notin Win[S_0]))$ **do**
$\quad e = (S, \alpha, S') \leftarrow pop(Waiting)$;
$\quad$ **if** $S' \notin Passed$ **then**
$\quad\quad Passed \leftarrow Passed \cup \{S'\}$;
$\quad\quad Depend[S'] \leftarrow \{(S, \alpha, S')\}$;
$\quad\quad Win[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
$\quad\quad Waiting \leftarrow Waiting \cup \{(S', \alpha, S'') \mid S'' = Post_\alpha(S')^{\nearrow}\}$;
$\quad\quad$ **if** $Win[S'] \neq \emptyset$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
$\quad$ **else (* reevaluate *)**
$\quad\quad Win^* \leftarrow Pred_t(Win[S] \cup \bigcup_{S \xrightarrow{c} T} cPred(Win[T]),$
$\quad\quad\quad \bigcup_{S \xrightarrow{u} T} uPred(T \setminus Win[T])) \cap S$;
$\quad\quad$ **if** $(Win[S] \subsetneq Win^*)$ **then**
$\quad\quad\quad Waiting \leftarrow Waiting \cup Depend[S]$; $Win[S] \leftarrow Win^*$;
$\quad\quad Depend[S'] \leftarrow Depend[S'] \cup \{e\}$;
$\quad$ **endif**
endwhile

Skip algorithm

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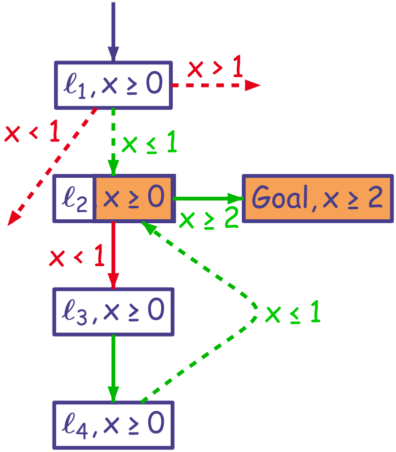

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  else (* reevaluate *)
    $\mathrm{Win}^* \leftarrow \mathrm{Pred}_t(\mathrm{Win}[S] \cup \bigcup_{S \xrightarrow{c} T} \mathrm{cPred}(\mathrm{Win}[T]), \bigcup_{S \xrightarrow{u} T} \mathrm{uPred}(T \setminus \mathrm{Win}[T])) \cap S$;
    if $(\mathrm{Win}[S] \subsetneq \mathrm{Win}^*)$ **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow \mathrm{Win}^*$;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

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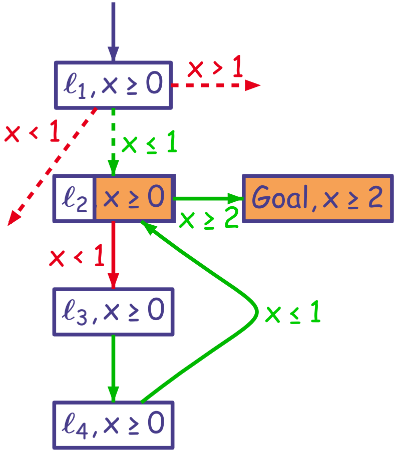

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  if $S' \notin$ Passed **then**
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    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    Win* $\leftarrow$Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
      $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
    if (Win$[S] \subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

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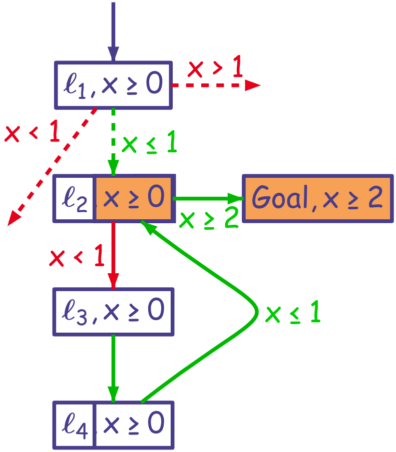

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    Win$^* \leftarrow$Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
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    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

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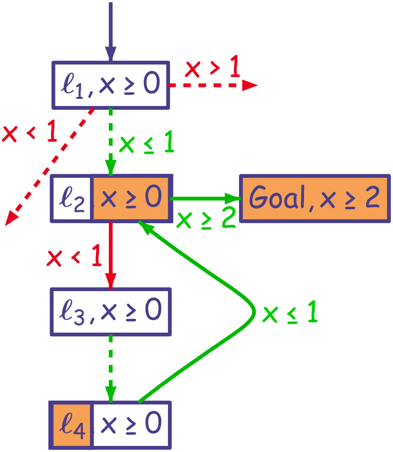

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Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
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Main:

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      $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
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      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

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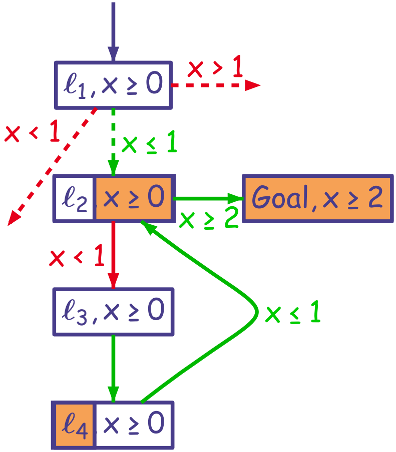

Skip algorithm

Initialization:

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      $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
    if (Win$[S] \subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

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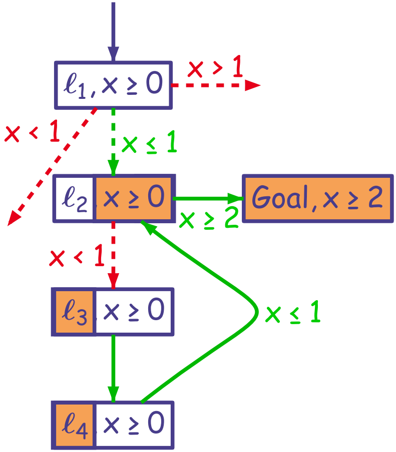

Initialization:

Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, \alpha, S') \mid S' = Post_\alpha(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

while $((\text{Waiting} \neq \emptyset) \wedge ((\ell_0, 0) \notin \text{Win}[S_0]))$ **do**
  $e = (S, \alpha, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
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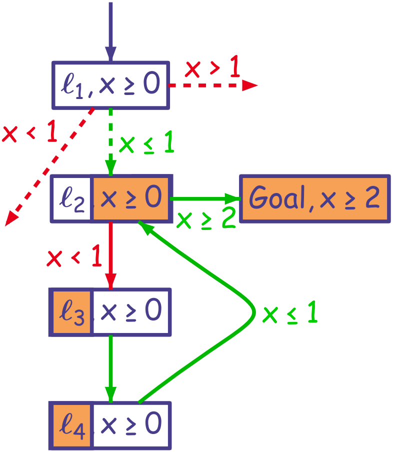
    $\bigcup_{S \xrightarrow{u} T} \text{uPred}(T \setminus \text{Win}[T])) \cap S$;
    if $(\text{Win}[S] \subsetneq \text{Win}^*)$ **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



Initialization:

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Win[S_0] $\leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend[S_0] $\leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ (($\ell_0, 0) \not\in$ Win[S_0])) **do**
  $e = (S, a, S') \leftarrow$ pop(Waiting);
  if $S' \not\in$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
    Depend[S'] $\leftarrow \{(S, a, S')\}$;
    Win[S'] $\leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
    Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
    if Win[S'] $\neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
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    Win* $\leftarrow \mathrm{Pred}_t(\mathrm{Win}[S] \cup \bigcup_{S \xrightarrow{c} T} \mathrm{cPred}(\mathrm{Win}[T]),$
      $\bigcup_{S \xrightarrow{u} T} \mathrm{uPred}(T \setminus \mathrm{Win}[T])) \cap S$;
    if (Win[S] $\subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend[S]; Win[S] $\leftarrow$ Win*;
    Depend[S'] $\leftarrow$ Depend[S'] $\cup \{e\}$;
  endif
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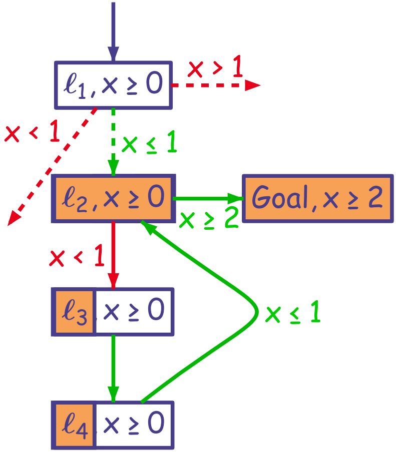
Skip algorithm

Liu & Smolka for Timed Games



Initialization:

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Waiting $\leftarrow \{(S_0, a, S') \mid S' = Post_a(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

```
while ((Waiting ≠ ∅) ∧ ((ℓ0, 0) ∉ Win[S0])) do
    e = (S, a, S') ← pop(Waiting);
    if S' ∉ Passed then
      Passed ← Passed ∪ {S'};
      Depend[S'] ← {(S, a, S')};
      Win[S'] ← S' ∩ ({Goal} × ℝ^X_≥0);
      Waiting ← Waiting ∪ {(S', a, S'') | S'' = Post_a(S')↗};
      if Win[S'] ≠ ∅ then Waiting ← Waiting ∪ {e};
    else (* reevaluate *)
      Win* ←Pred_t(Win[S] ∪ ⋃_{S -c-> T} cPred(Win[T]),
                            ⋃_{S -u-> T} uPred(T \ Win[T])) ∩ S;
      if (Win[S] ⊊ Win*) then
        Waiting ← Waiting ∪ Depend[S]; Win[S] ← Win*;
      Depend[S'] ← Depend[S'] ∪ {e};
    endif
endwhile
```

Skip algorithm

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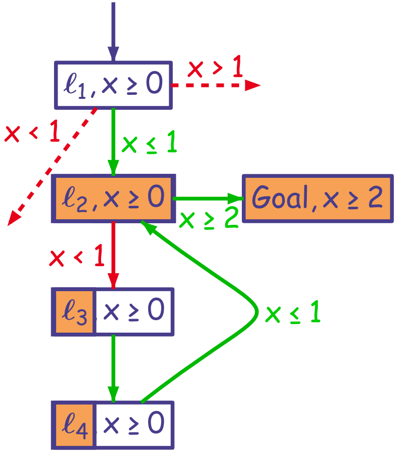

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Depend$[S_0] \leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ (($\ell_0, 0) \notin$ Win$[S_0]$)) **do**
$e = (S, a, S') \leftarrow$ pop(Waiting);
if $S' \notin$ Passed **then**
Passed $\leftarrow$ Passed $\cup \{S'\}$;
Depend$[S'] \leftarrow \{(S, a, S')\}$;
Win$[S'] \leftarrow S' \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Waiting $\leftarrow$ Waiting $\cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
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Win$^* \leftarrow$Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
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if (Win$[S] \subsetneq$ Win*) **then**
Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
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endif
endwhile

▸▸ Skip algorithm

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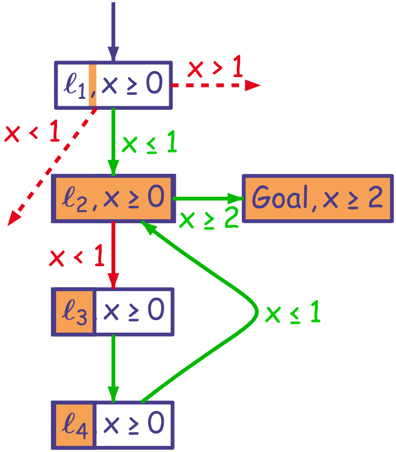

Initialization:

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      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
    Depend$[S'] \leftarrow$ Depend$[S'] \cup \{e\}$;
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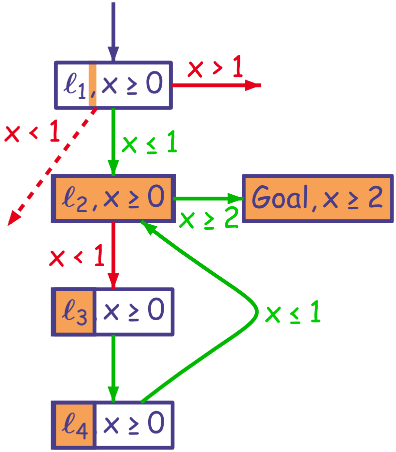
Skip algorithm

Liu & Smolka for Timed Games



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Passed $\leftarrow \{S_0\}$ **where** $S_0 = \{(\ell_0, 0)\}^{\nearrow}$;
Waiting $\leftarrow \{(S_0, \alpha, S') \mid S' = Post_\alpha(S_0)^{\nearrow}\}$;
Win$[S_0] \leftarrow S_0 \cap (\{Goal\} \times \mathbb{R}^X_{\geq 0})$;
Depend$[S_0] \leftarrow \emptyset$;

Main:

while ((Waiting $\neq \emptyset$) $\wedge$ ((ℓ_0, 0) $\notin$ Win$[S_0]$)) **do**
  $e = (S, \alpha, S') \leftarrow$ pop(Waiting);
  if $S' \notin$ Passed **then**
    Passed $\leftarrow$ Passed $\cup \{S'\}$;
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    if Win$[S'] \neq \emptyset$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
  else (* reevaluate *)
    Win$^* \leftarrow$Pred$_t$(Win$[S] \cup \bigcup_{S \xrightarrow{c} T}$ cPred(Win$[T]$),
      $\bigcup_{S \xrightarrow{u} T}$ uPred($T \setminus$ Win$[T]$)) $\cap S$;
    if (Win$[S] \subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend$[S]$; Win$[S] \leftarrow$ Win*;
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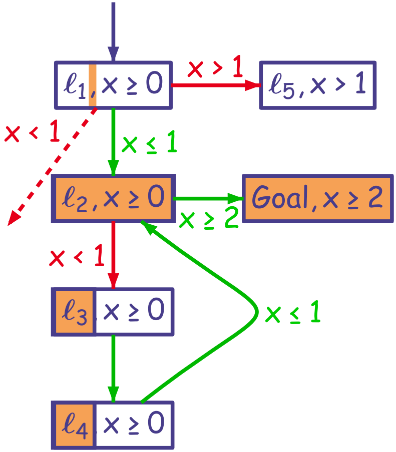
  endif
endwhile

Skip algorithm

Liu & Smolka for Timed Games



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$Waiting \leftarrow Waiting \cup \{(S', a, S'') \mid S'' = Post_a(S')^{\nearrow}\}$;
if $Win[S'] \neq \emptyset$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
else (* reevaluate *)
$Win^* \leftarrow Pred_t(Win[S] \cup \bigcup_{S \xrightarrow{c} T} cPred(Win[T]),$
$\bigcup_{S \xrightarrow{u} T} uPred(T \setminus Win[T])) \cap S$;
if $(Win[S] \subsetneq Win^*)$ **then**
$Waiting \leftarrow Waiting \cup Depend[S]$; $Win[S] \leftarrow Win^*$;
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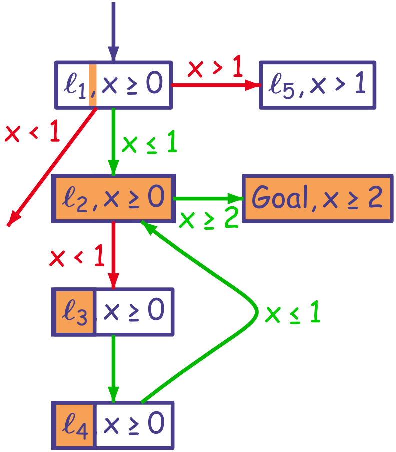
endwhile

Skip algorithm

Liu & Smolka for Timed Games



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      $\bigcup_{S \xrightarrow{u} T}$ uPred(T \ Win[T])) $\cap$ S;
    if (Win[S] $\subsetneq$ Win*) **then**
      Waiting $\leftarrow$ Waiting $\cup$ Depend[S]; Win[S] $\leftarrow$ Win*;
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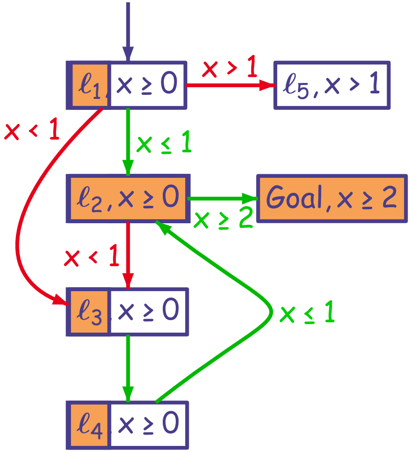
Skip algorithm

Liu & Smolka for Timed Games



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Skip algorithm

Summary of the Results [Concur'05]

- A True on-the-fly algorithm for reachability control
- Winning Strategies can be computed
- Termination
  A symbolic edge (S, a, T) will be at most (1+ # regions(T)) times in *Waiting* list
- Complexity
  A region may be in many symbolic states
  Our algorithm: Not linear in the size of the region graph hence not theoretically optimal
- However ... seems good in practice with **UPPAAL-TiGA**

Download at **`http://www.cs.aau.dk/~adavid/tiga/`**

Next:

- Reachability Control
- On-the-fly Algorithms for Reachability Control
- **Implementation, Optimizations, Time Optimality**
- Partial Observation

Efficient Implementation of Pred_t

Theorem

The following distribution law holds:

$$\mathsf{Pred}_t(\bigcup_i G_i, \bigcup_j B_j) = \bigcup_i \bigcap_j \mathsf{Pred}_t(G_i, B)$$

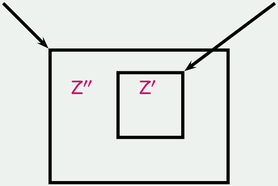
Theorem

If B is a convex set, then:

$$\mathsf{Pred}_t(G, B) = (G^{\swarrow} \setminus B^{\swarrow}) \cup ((G \cap B^{\swarrow}) \setminus B)^{\swarrow}$$

Inclusion Checking & Losing States

Inclusion checking



Losing States

Pruning

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ ($s_0 \notin$ Win[S_0])) **do**
    $e = (S, a, S') \longleftarrow$ pop(Waiting);
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            (...)
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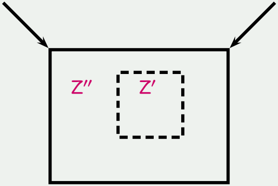

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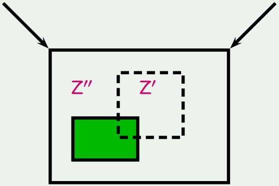

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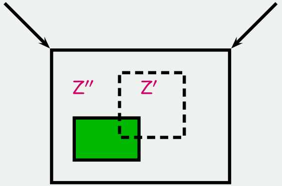

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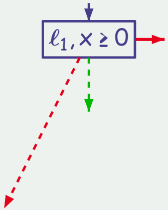

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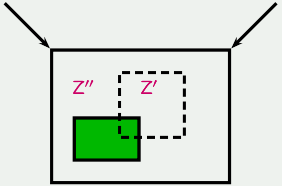

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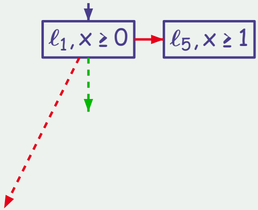
Losing States



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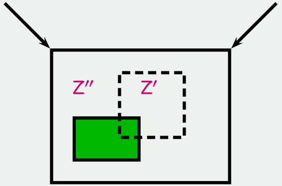

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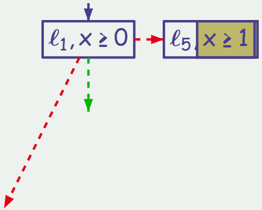

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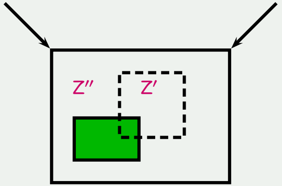

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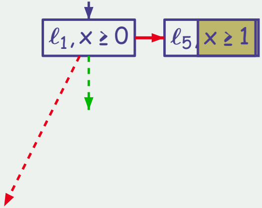

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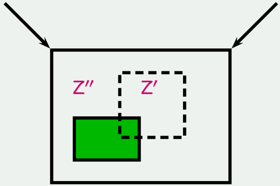

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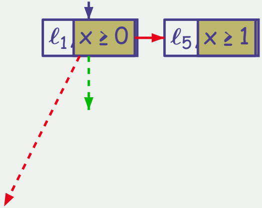

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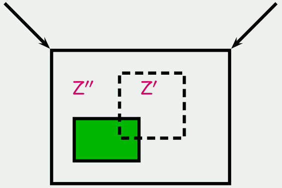

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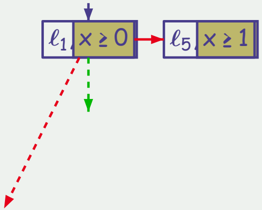

Pruning

Main:
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    e = (S, a, S′) ← pop(Waiting);
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            (. . .)
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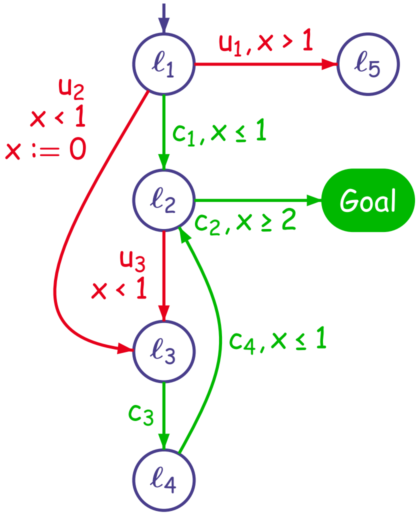
Time Optimality for Free



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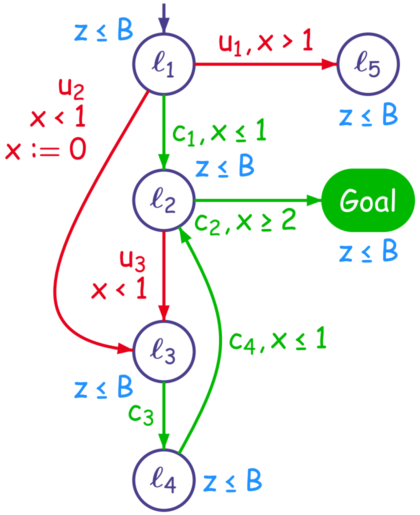
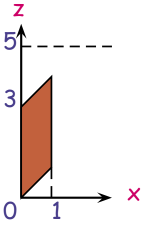


Assume:

- The initial state is winning
- We know an upper bound B of the (optimal) time needed to reach Goal

To compute the optimal time:

- Add a clock z (unconstrained at the beginning)
- Add a global invariant $z \leq B$



Next:

- Reachability Control
- On-the-fly Algorithms for Reachability Control
- Implementation, Optimizations, Time Optimality
- Partial Observation
  - Results About Partial Observation for Timed Games
  - Stuttering Free Observations
  - Knowledge Based Subset Construction

Results for Partial Observation

Partial Observation of Events:

- Discrete Event Systems:
  Partial Obs. = Hiding + Determinization + Full Observation
  [Kupferman & Vardi'99, Reif'84, Arnold et al.'03]
- Timed Automata:
  "Invisible" ε-transition cannot be removed [Bérard et al.'98]
  Timed Automata cannot be determinized [Alur & Dill'94]

Theorem ([Bouyer et al.'03])

Safety and reachability control under partial observation are undecidable.

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Control under partial observation with bounded resources is 2EXPTIME-complete.

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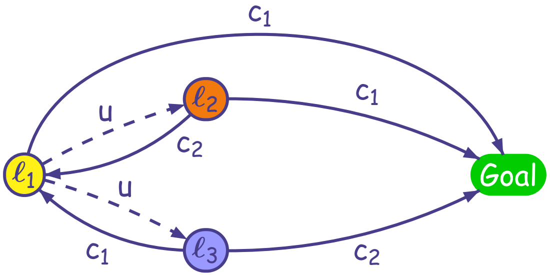
State-Based Partial Observation



- **Full Observation**: in ℓ_2 do c_1, in ℓ_3 do c_2
- Partial Observation: Partition of the state space e.g. $\ell_2 \equiv \ell_3$ [Chatterjee et al.'06, De Wulf et al.'06]

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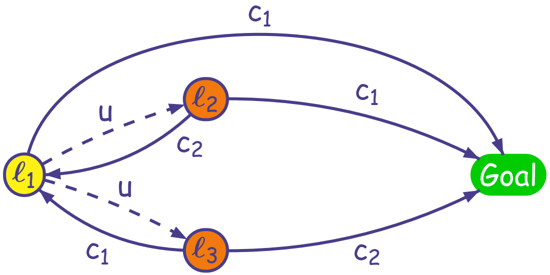

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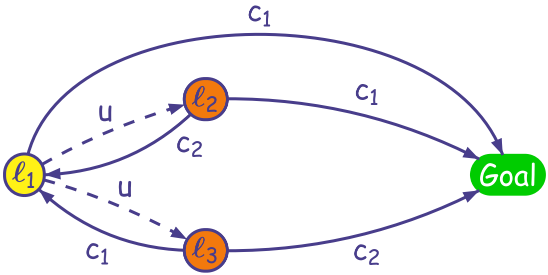

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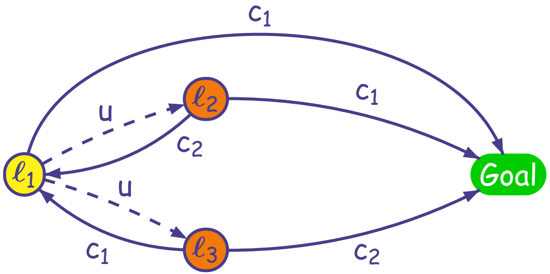
Impossible to Win

State-Based Partial Observation



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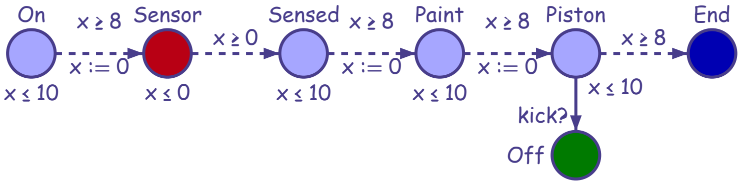
Our Aim:

- Extend this framework to timed systems
- Design efficient algorithms for solving this type of games

Example



$$(On,0) \xrightarrow{8} (Sensor,0) \longrightarrow (Sensed,0) \xrightarrow{9} (Paint,0) \xrightarrow{8.7} (Piston,0) \xrightarrow{kick?} Off$$

$$\bullet \xrightarrow{8} \bullet \longrightarrow \bullet \xrightarrow{9} \bullet \xrightarrow{8.7} \bullet \xrightarrow{kick?} \bullet$$

Assumption: the controller can only see changes of observations

Stuttering-Free observation: $\bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet$

Must play based on stuttering-free observations

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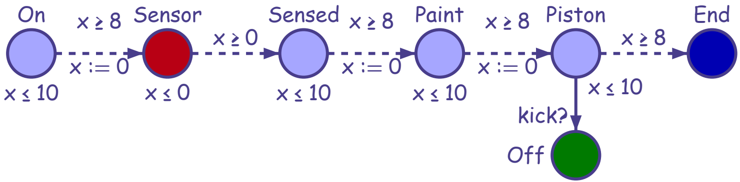

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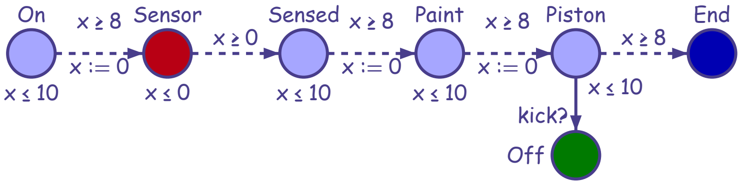

$$(\mathrm{On},0) \xrightarrow{8} (\mathrm{Sensor},0) \longrightarrow (\mathrm{Sensed},0) \xrightarrow{9} (\mathrm{Paint},0) \xrightarrow{8.7} (\mathrm{Piston},0) \xrightarrow{kick?} \mathrm{Off}$$

$$\bullet \xrightarrow{8} \bullet \longrightarrow \bullet \xrightarrow{9} \bullet \xrightarrow{8.7} \bullet \xrightarrow{kick?} \bullet$$

Assumption: the controller can only see changes of observations

Stuttering-Free observation: $\bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet$

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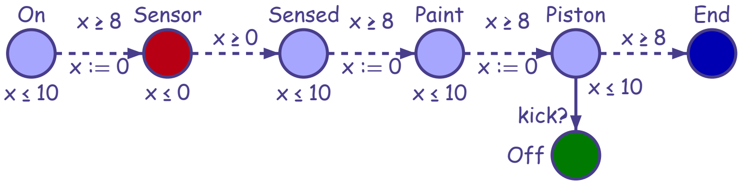

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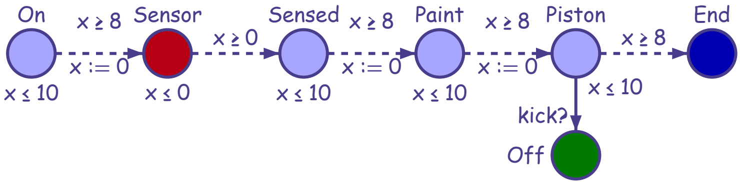

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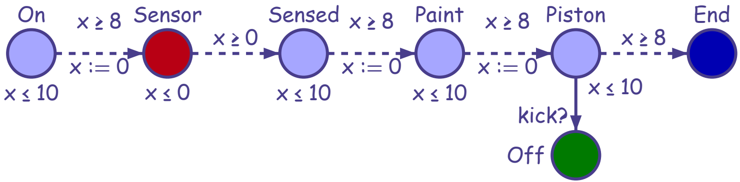

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Rules of the Game

Given:

- a TGA automaton G
- a finite set of observations $\mathcal{O}$ (maps states to observations)
- a control objective $\Phi \subseteq \mathcal{O}^\omega$

Observation-Based Stuttering Invariant Strategies

f is a OBSI strategy if:

$$\text{Observation}(\rho) \equiv_{\text{stutt}} \text{Observation}(\rho') \text{ implies } f(\rho) = f(\rho')$$

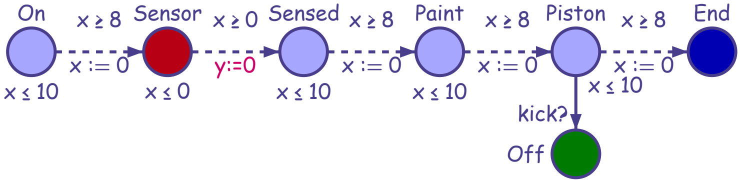
Control under Partial Observation:

Is there an OBSI winning strategy ?

Requirements:

- Observations have special shape:
  must become true at some first instant / partition the state space
  e.g. $20 \leq y < 24$
- Φ is stuttering closed

Knowledge Based Subset Construction



Observations = colors + $y < 20$, $20 \leq y < 24$ and $24 \leq y$

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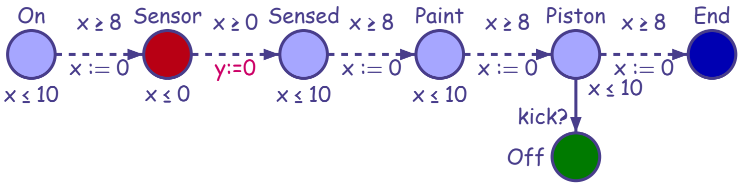

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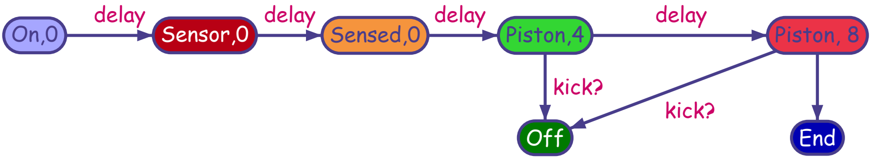

Result for Partial Observation

Input:

- a TGA automaton G
- a stuttering closed control objective Φ

Knowledge Game:

1. Build a Knowledge Game Know(G)
   + take care of infinite single-observation runs
2. if G is bounded Know(G) is finite

Theorem

The controller has a winning strategy in (Know(G), Φ) iff it has an OBSI winning strategy in (G, Φ).

Holds for any control objective

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Efficient Algorithm for Reachability Game

Initialization:
Passed $\leftarrow \{q_0\}$;
Waiting $\leftarrow \{(q_0, a, q') \mid a \in Act\ q \xrightarrow{a} q'\}$;
Win$[q_0] \leftarrow (q_0 \in Goal\ ?\ 1 : 0)$;
Depend$[q_0] \leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ Win$[q_0] \neq 1$)) **do**
$e = (q, a, q') \leftarrow$ pop(Waiting);
if $q' \notin$ Passed **then** {
Passed $\leftarrow$ Passed $\cup \{q'\}$;
Depend$[q'] \leftarrow \{(q, a, q')\}$;
Win$[q'] \leftarrow (q' \in Goal\ ?\ 1 : 0)$;
Waiting $\leftarrow$ Waiting $\cup \{(q', a, q'') \mid q' \xrightarrow{a} q''\}$;
Win$^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
if Win$[q']$ **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
}
else (* **reevaluate** *)
Win$^*[q] \leftarrow$ **Update**(Win$^*[q]$) ;
if (Win$^*[q] = (k, 0) \wedge k \geq 1$) **then** {
Waiting $\leftarrow$ Waiting $\cup$ Depend$[q]$;
Win$[q] \leftarrow 1$;
}
if Win$[q'] = 0$ **then**
Depend$[q'] \leftarrow$ Depend$[q'] \cup \{e\}$;
endif
endwhile

Initialization:
Passed $\leftarrow \{\{s_0\}\}$;
Waiting $\leftarrow \{(\{s_0\}, a, W') \mid a \in \Sigma_1,\ o \in \mathcal{O},\ \ W' = \text{Next}_a(\{s_0\}) \cap o \wedge W' \neq \emptyset\}$;
Win$[\{s_0\}] \leftarrow (\{s_0\} \subseteq \gamma(\text{Goal})\ ?\ 1 : 0)$;
Losing$[\{s_0\}] \leftarrow (\{s_0\} \not\subseteq \gamma(\text{Goal}) \wedge (\text{Waiting} = \emptyset \vee \forall a \in \Sigma_1, \text{Sink}_a(s_0) \neq \emptyset)\ ?\ 1 : 0)$;
Depend$[\{s_0\}] \leftarrow \emptyset$;

Main:
while ((Waiting $\neq \emptyset$) $\wedge$ Win$[\{s_0\}] \neq 1 \wedge$ Losing$[\{s_0\}] \neq 1$)) **do**
$e = (W, a, W') \leftarrow$ pop(Waiting);
if $s' \notin$ Passed **then**
Passed $\leftarrow$ Passed $\cup \{W'\}$;
Depend$[W'] \leftarrow \{(W, a, W')\}$;
Win$[W'] \leftarrow (W' \subseteq \gamma(\text{Goal})\ ?\ 1 : 0)$;
Losing$[W'] \leftarrow (W' \not\subseteq \gamma(\text{Goal}) \wedge \text{Sink}_a(W') \neq \emptyset\ ?\ 1 : 0)$;
if (Losing$[W'] \neq 1$) **then** (* **if losing it is a deadlock state** *)
NewTrans $\leftarrow \{(W', a, W'') \mid a \in \Sigma,\ o \in \mathcal{O},\ \ W' = \text{Next}_a(W) \cap o \wedge W' \neq \emptyset\}$;
if NewTrans $= \emptyset \wedge$ Win$[W'] = 0$ **then** Losing$[W'] \leftarrow 1$;
if (Win$[W'] \vee$ Losing$[W']$) **then** Waiting $\leftarrow$ Waiting $\cup \{e\}$;
Waiting $\leftarrow$ Waiting $\cup$ NewTrans;
else (* **reevaluate** *)
Win$^* \leftarrow \bigvee_{c \in \text{Enabled}(W)} \bigwedge_{W \xrightarrow{c} W''}$ Win$[W'']$;
if Win* **then**
Waiting $\leftarrow$ Waiting $\cup$ Depend$[W]$; Win$[W] \leftarrow 1$;
Losing$^* \leftarrow \bigwedge_{c \in \text{Enabled}(W)} \bigvee_{W \xrightarrow{c} W''}$ Losing$[W'']$;
if Losing* **then**
Waiting $\leftarrow$ Waiting $\cup$ Depend$[W]$; Losing$[W] \leftarrow 1$;
if (Win$[W'] = 0 \wedge$ Losing$[W'] = 0$) **then** Depend$[W'] \leftarrow$ Depend$[W'] \cup \{e\}$;
endif
endwhile

Conclusion & Ongoing Work

Conclusion

- Efficient algorithm for reachability timed games
  Extends to safety games
- Implementation in UPPAAL-TiGA
  http://www.cs.aau.dk/~adavid/tiga/
- Experiments: Pigs Factory + Jobshop scheduling

Ongoing Work

- On-the-fly algorithm for Büchi objectives
- Compact representation of winning strategies
- Extension to optimal cost computation (reachability)
- Simulation of strategies in UPPAAL-TiGA

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Timed Automata

[Alur & Dill'94]

A Timed Automaton $\mathcal{A}$ is a tuple $(L, \ell_0, Act, X, \mathrm{inv}, \longrightarrow)$ where:

- L is a finite set of locations
- ℓ_0 is the initial location
- X is a finite set of clocks
- Act is a finite set of actions
- $\longrightarrow$ is a set of transitions of the form $\ell \xrightarrow{g,a,R} \ell'$ with:
  - $\ell, \ell' \in L$,
  - $a \in Act$
  - a guard g which is a clock constraint over X
  - a reset set R which is the set of clocks to be reset to 0

Clock constraints are boolean combinations of $x \sim k$ with $x \in C$ and $k \in \mathbb{Z}$ and $\sim \in \{\leq, <\}$.

Semantics of Timed Automata

Let $\mathcal{A} = (L, \ell_0, Act, X, inv, \longrightarrow)$ be a Timed Automaton.

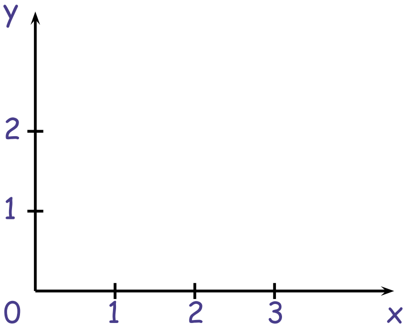
A **state** (ℓ, v) of $\mathcal{A}$ is in $L \times \mathbb{R}_{\geq 0}^X$

The semantics of $\mathcal{A}$ is a **Timed Transition System** $S_{\mathcal{A}} = (Q, q_0, Act \cup \mathbb{R}_{\geq 0}, \longrightarrow)$ with:

- $Q = L \times \mathbb{R}_{\geq 0}^X$
- $q_0 = (\ell_0, \overline{0})$
- $\longrightarrow$ consists in:

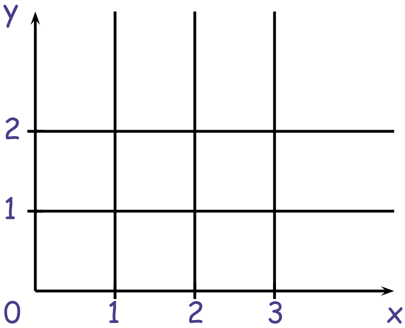
discrete transition: $(\ell, v) \xrightarrow{a} (\ell', v') \iff \begin{cases} \exists\, \ell \xrightarrow{g,a,r} \ell' \in \mathcal{A} \\ v \models g \\ v' = v[r \leftarrow 0] \\ v' \models inv(\ell') \end{cases}$

delay transition: $(\ell, v) \xrightarrow{d} (\ell, v + d) \iff d \in \mathbb{R}_{\geq 0} \wedge v + d \models inv(\ell)$


y
2
1
0
1
2
3
x

The Region Abstraction

[Alur & Dill'94]



Build an equivalence relation which is of finite index and is:

- "compatible" with clock constraints $(g ::= x \sim c \quad g \wedge g)$

  $r, r' \in R \implies \forall$ constraints $g, \quad r \models g \iff r' \models g$

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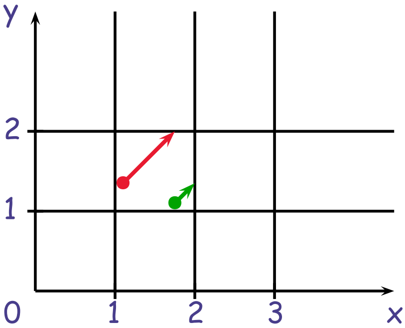

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- "compatible" with time elapsing
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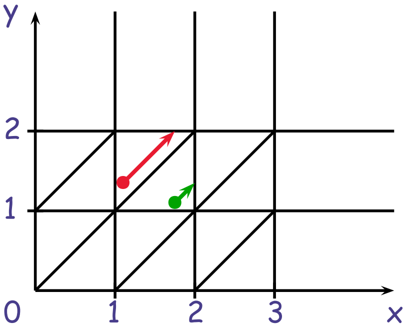

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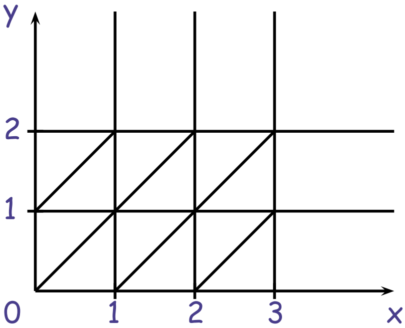

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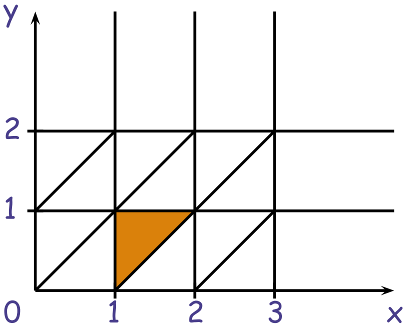

region defined by
$I_x =]1;2[\; I_y =]0;1[$
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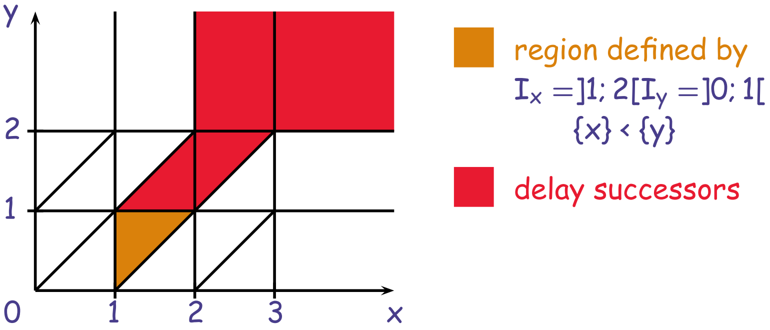

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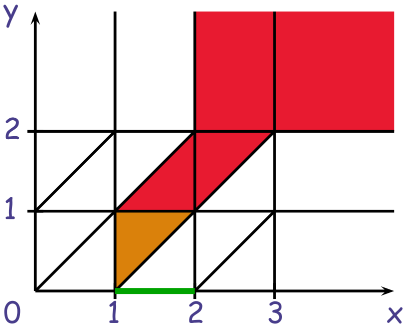

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delay successors

successor by reset

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The Region Automaton

- For each transition $\ell \xrightarrow{g,a,C:=0} \ell'$ of the TA
- Build transitions in the region automaton RA: $(\ell, R) \xrightarrow{a} (\ell', R')$ if:
  - there exists R'' a delay successor of R s.t.
  - R'' satisfies the guard g ($R'' \subseteq [\![g]\!]$)
  - $R''[C \leftarrow 0]$ is included in R'

a TA and its region automaton RA are time-abstract bisimilar

- The region automaton is finite
- Language accepted by the RA = untimed language accepted by the TA

  a timed word $w = (a, 1.2)(b, 3.4)(a, 6.256)$; $untimed(w) = aba$
- Language Emptyness can be decided on the RA

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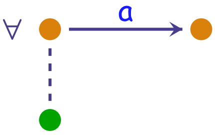

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\begin{array}{ccccccc}
(\ell_0, v_0) & \xrightarrow{a_1, t_1} & (\ell_1, v_1) & \xrightarrow{a_2, t_2} & (\ell_2, v_2) & \xrightarrow{a_3, t_3} & \dots \\
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with $v_i \in R_i$ for all i.

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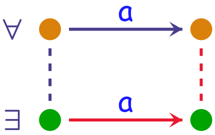

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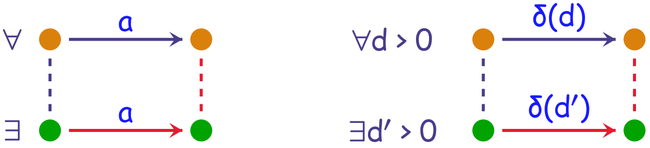

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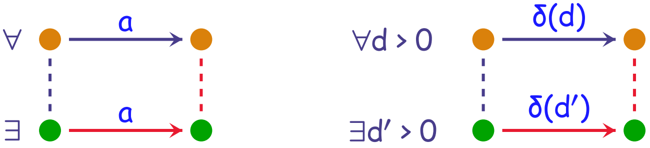

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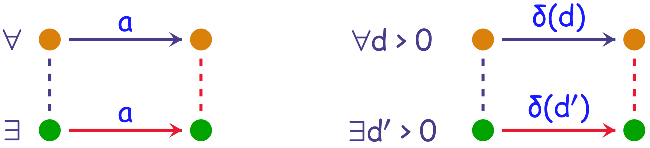

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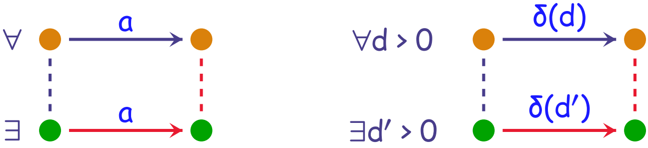

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\begin{tikzcd}
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Definition (Outcome in Timed Games)

Let $G = (L, \ell_0, Act, X, E, inv)$ be a TGA and f a strategy over G. The **outcome** $\mathrm{Outcome}((\ell, v), f)$ of f from configuration (ℓ, v) in G is the subset of $\mathrm{Runs}((\ell, v), G)$ defined inductively by:

- $(\ell, v) \in \mathrm{Outcome}((\ell, v), f)$,
- if $\rho \in \mathrm{Outcome}((\ell, v), f)$ then $\rho' = \rho \xrightarrow{e} (\ell', v') \in \mathrm{Outcome}((\ell, v), f)$ if $\rho' \in \mathrm{Runs}((\ell, v), G)$ and one of the following three conditions hold:
  1. $e \in Act_u$,
  2. $e \in Act_c$ and $e = f(\rho)$,
  3. $e \in \mathbb{R}_{\geq 0}$ and $\forall 0 \leq e' < e, \exists (\ell'', v'') \in (L \times \mathbb{R}^X_{\geq 0})$ s.t. $last(\rho) \xrightarrow{e'} (\ell'', v'') \wedge f(\rho \xrightarrow{e'} (\ell'', v'')) = \lambda$.
- an infinite run ρ is in $\in \mathrm{Outcome}((\ell, v), f)$ if all the finite prefixes of ρ are in $\mathrm{Outcome}((\ell, v), f)$.

States & Symbolic States

- $Q = L \times \mathbb{R}_{\geq 0}^{Clock}$ is the set of states of the TGA
  $$q = (\ell, v) \in Q$$
- Discrete predecessors of $X \subseteq Q$ by an action a:
  $$\mathrm{Pred}^a(X) = \{q \in Q \mid q \xrightarrow{a} q' \text{ and } q' \in X\}$$
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- Zone = conjunction of triangular constraints
  $x - y < 3, x \geq 2 \wedge 1 < y - x < 2$
- Symbolic State is defined by a State predicate (SP)
  $P = \cup_{i \in [1..n]}(\ell_{j_i}, Z_i)$, $\ell_{j_i} \in L$, Z_i is a zone
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Effectiveness of Pred^a and Pred^δ

If P is a SP then $\mathrm{Pred}^a(P), \mathrm{Pred}^\delta(P)$ are SP and can be computed effectively. (There is a symbolic version for Pred^a and Pred^δ.)

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Symbolic Computation For Timed Games

X is a state predicate

- $cPred(X) = \bigcup_{c \in Act_c} Pred^c(X)$ $uPred(X) = \bigcup_{u \in Act_u} Pred^u(X)$
  cPred and uPred are effectively computable
- $Pred_\delta(X, Y)$: Time controllable predecessors of X avoiding Y:

q $q' \in X$

$Pred_\delta(X, Y)$ is effectively computable for state predicates X, Y

- Controllable Predecessors Operator for Timed Games

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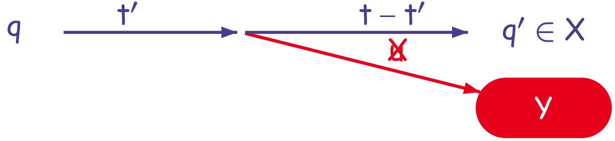

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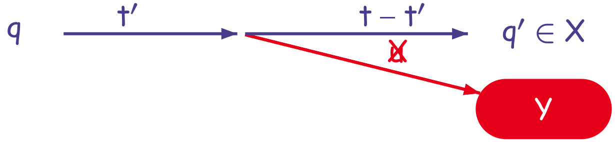

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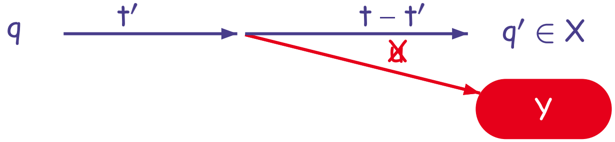

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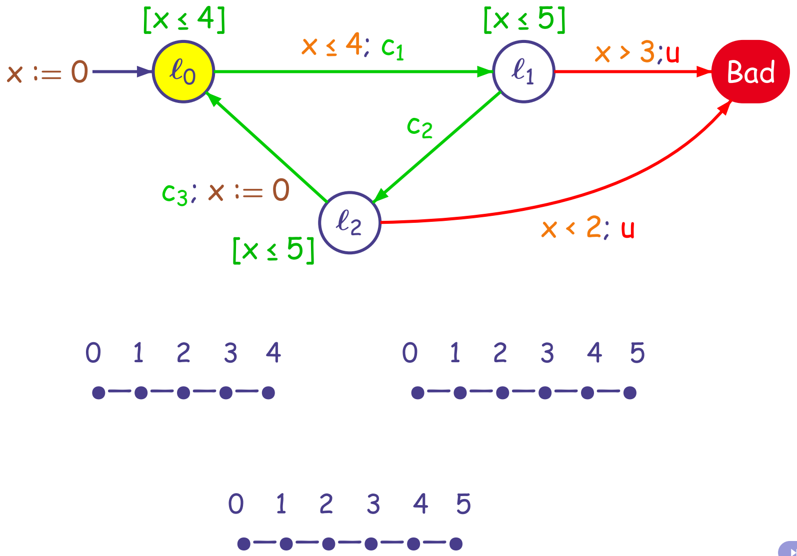

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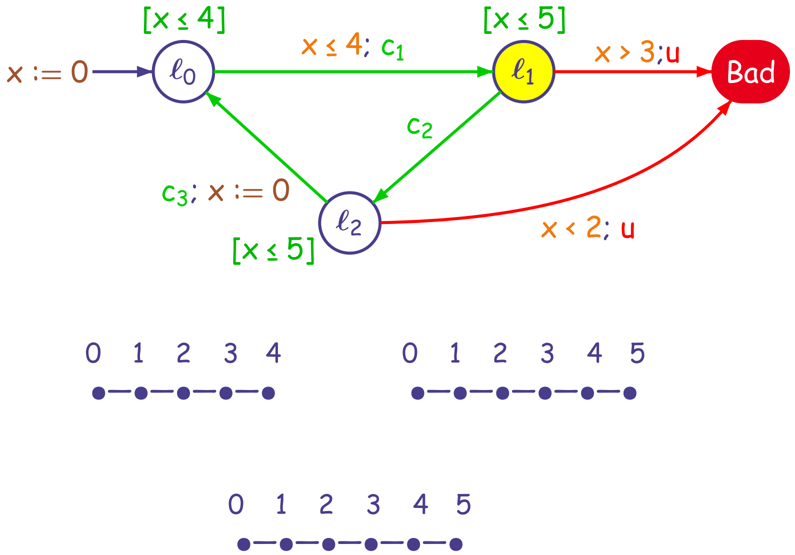

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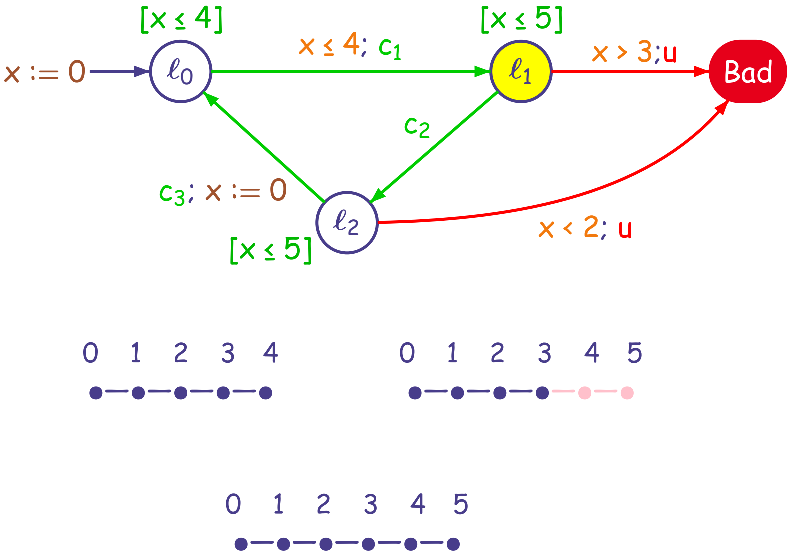

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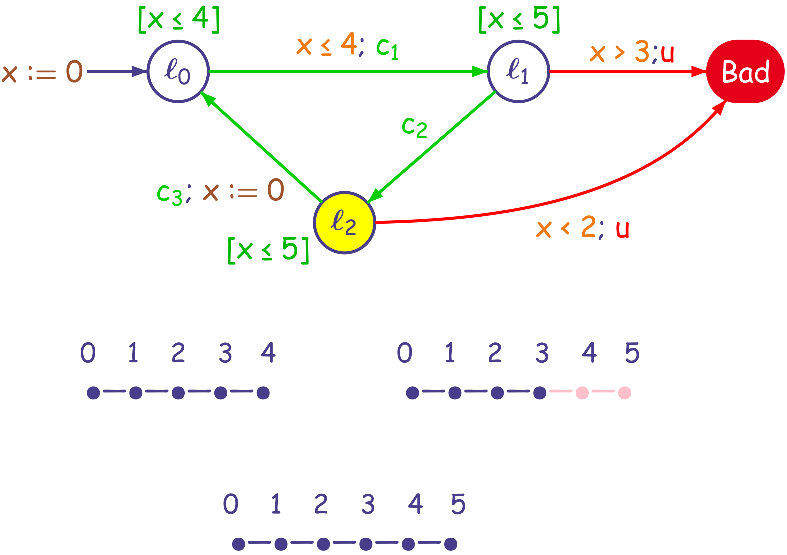

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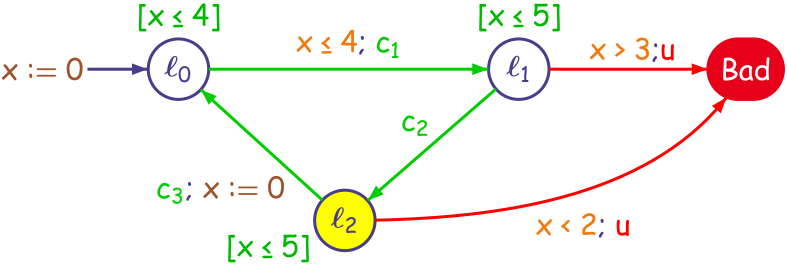

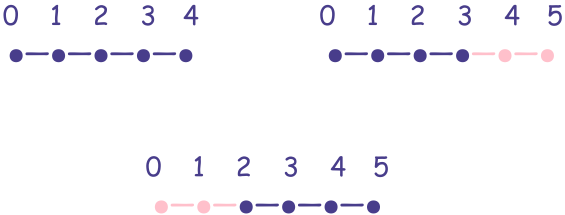

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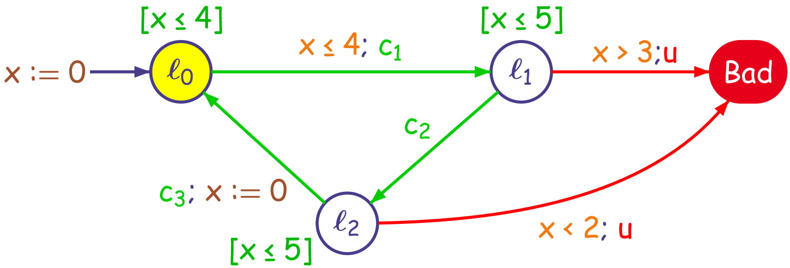

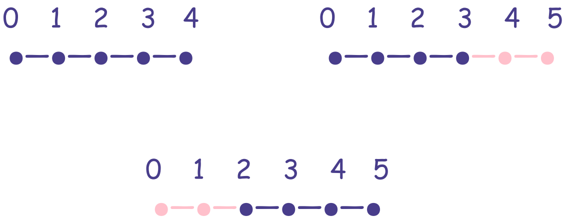

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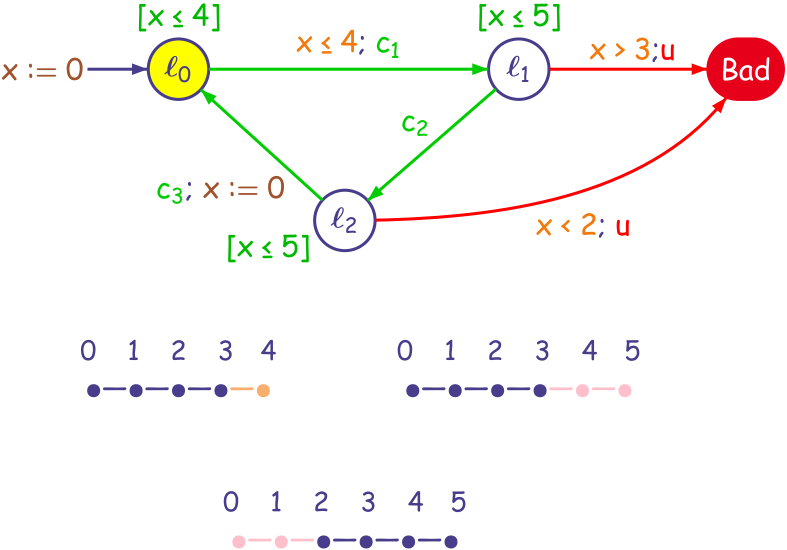

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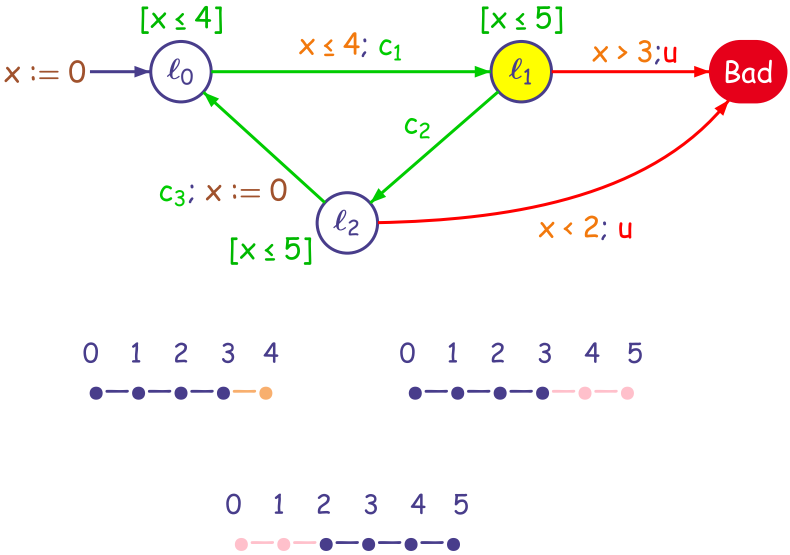

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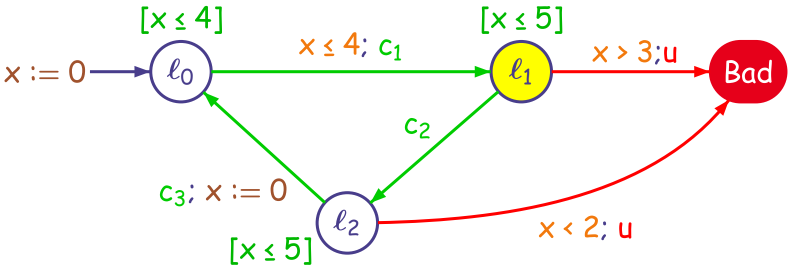

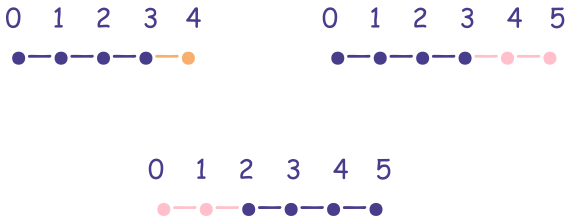

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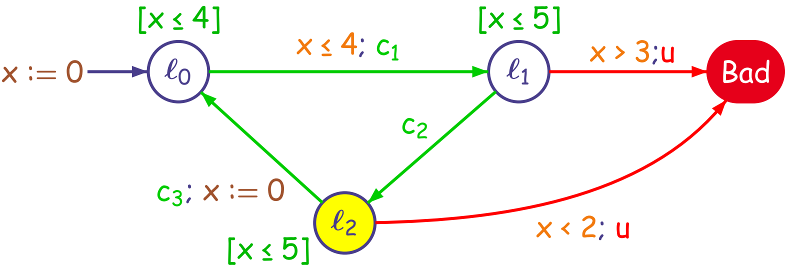

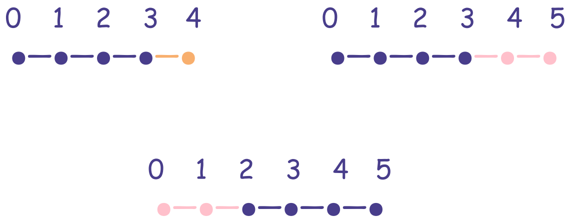

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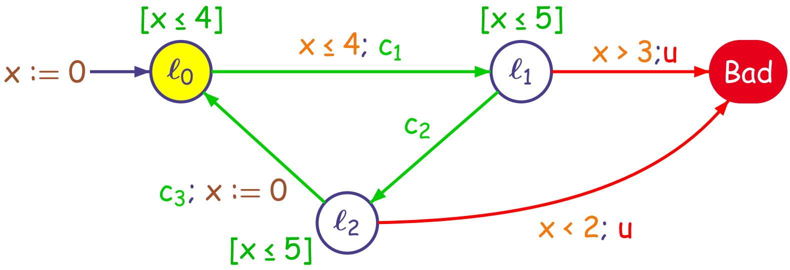

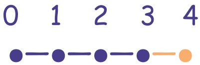

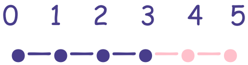

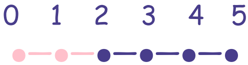

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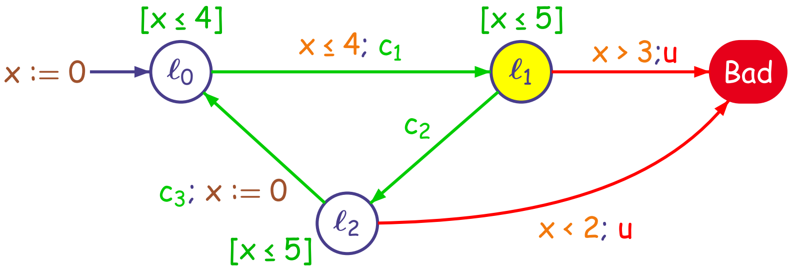

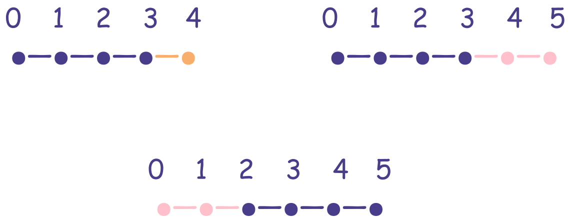

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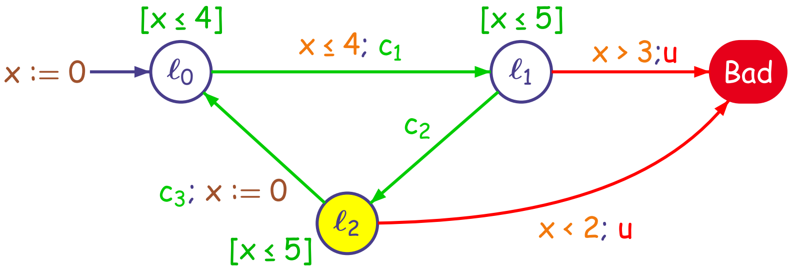

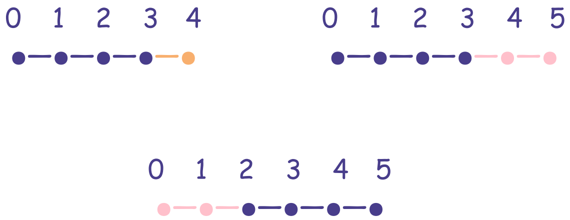

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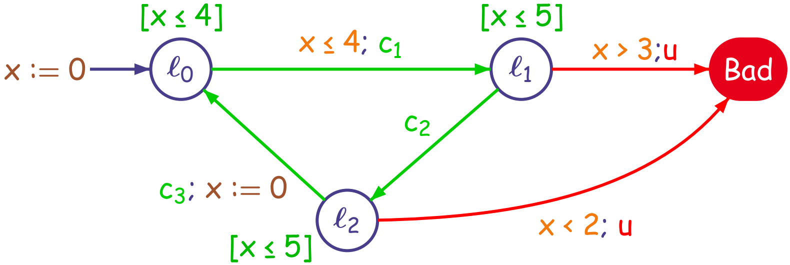
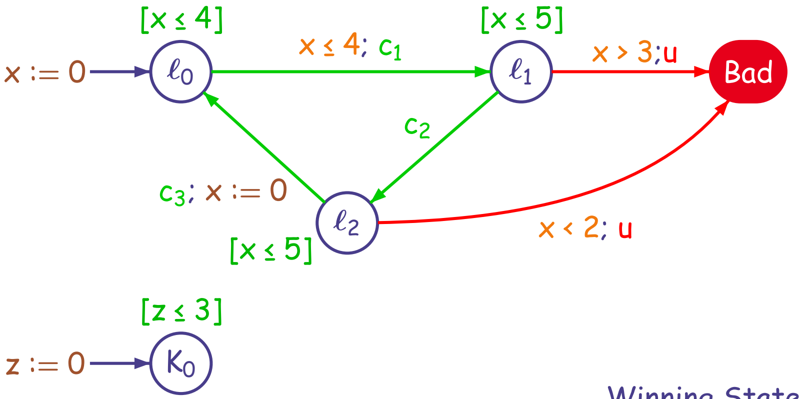


Winning States

$(\ell_0, 0 \leq x \leq 3)$
$(\ell_1, 0 \leq x \leq 3)$
$(\ell_2, 2 \leq x \leq 5)$

Example of Computation for Safety Games



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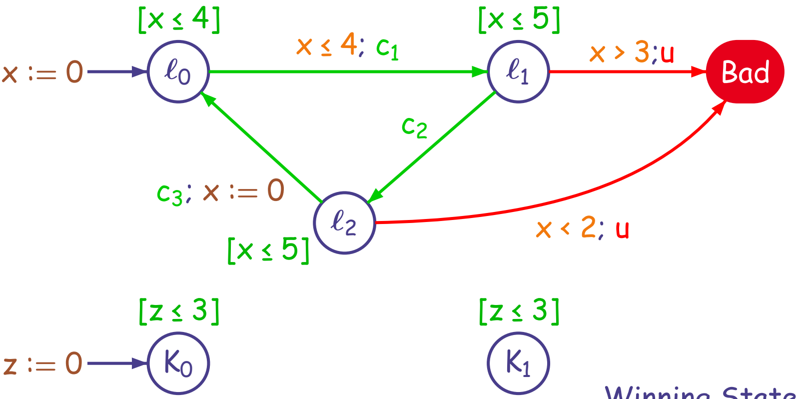

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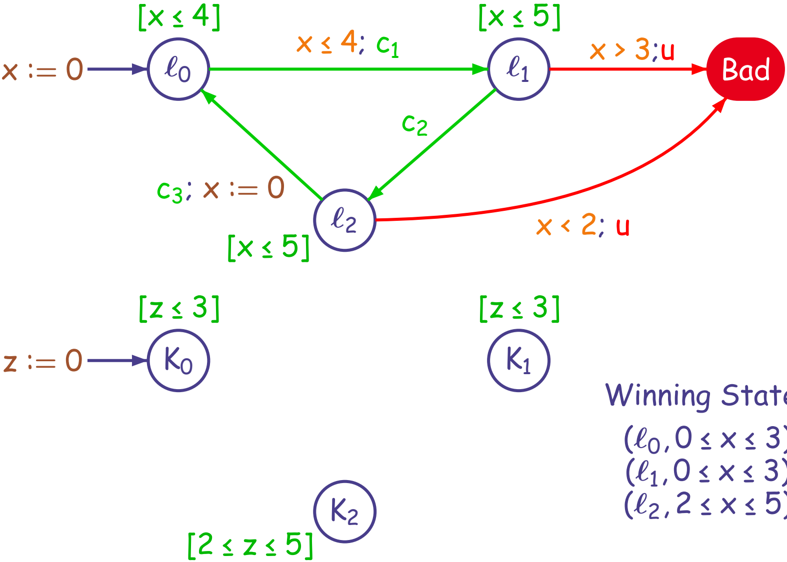

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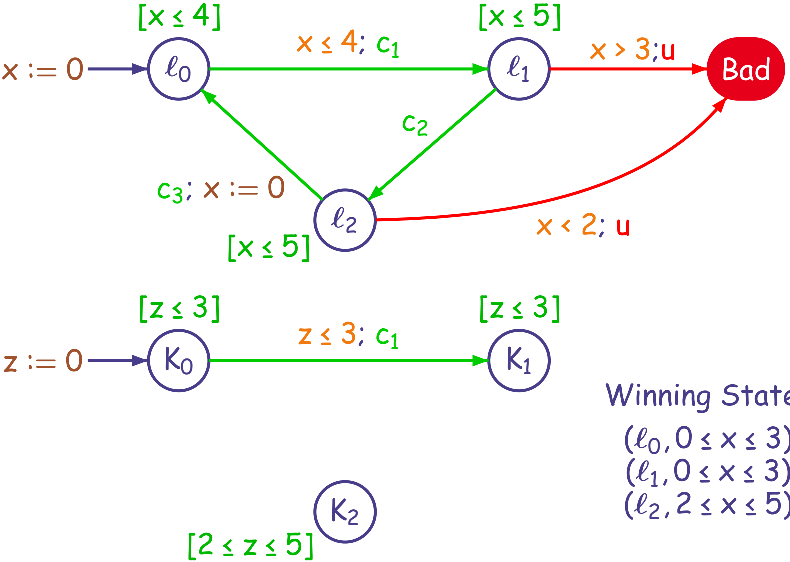

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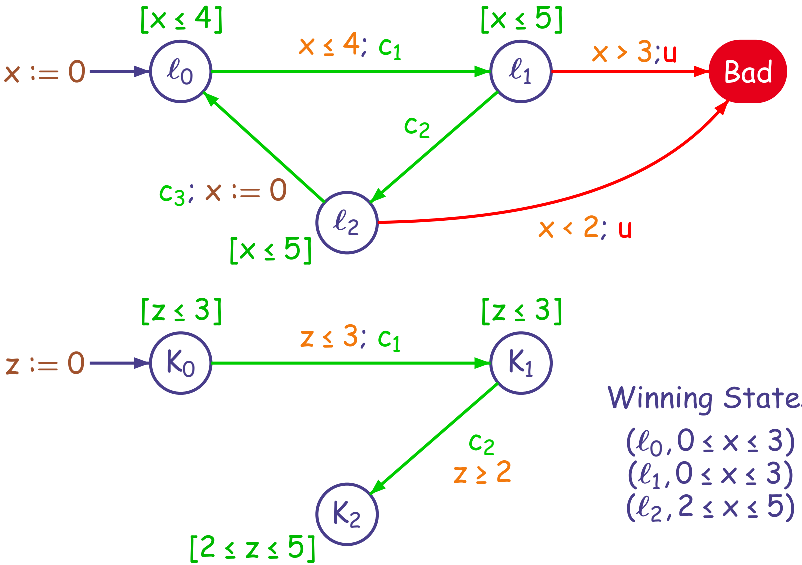

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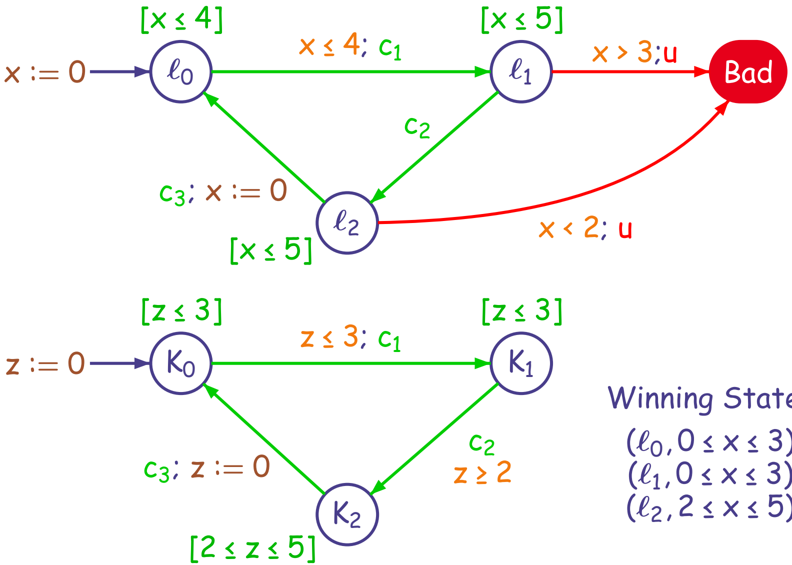

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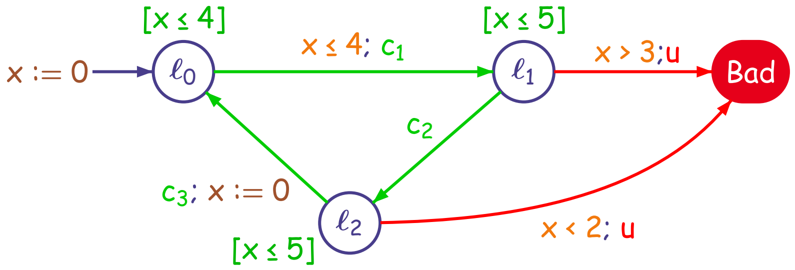

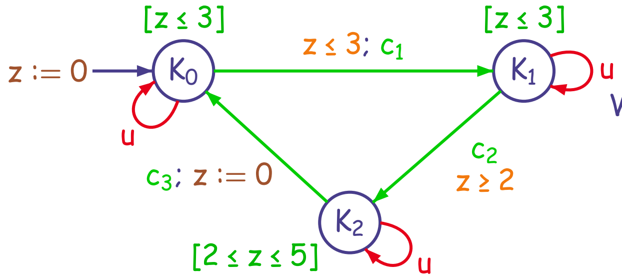

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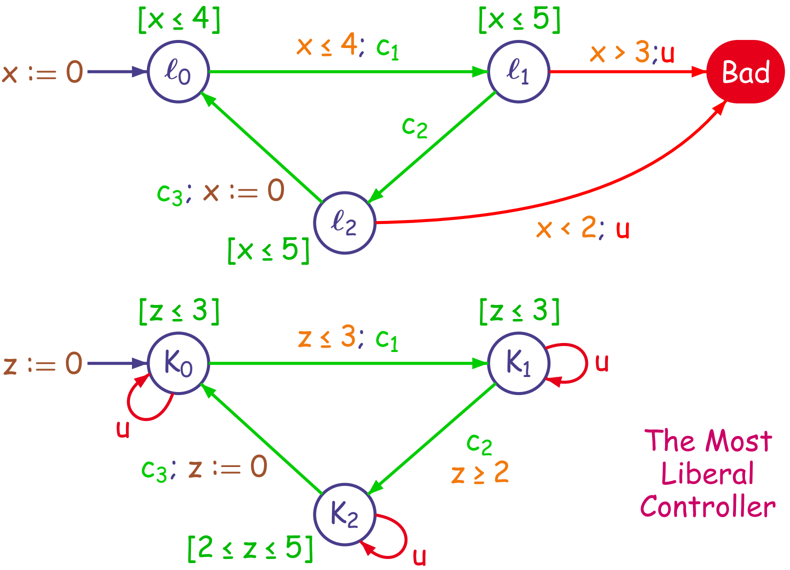

The Most Liberal Controller