Efficient On-the-fly Algorithms for the Analysis of Timed Games

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August 23, 2005

Outline

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Control Problems

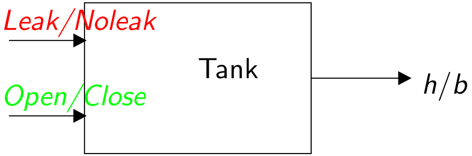
Reachability Control

On-the-fly Algorithms for Reachability Control

Implementation, Optimizations, Time Optimality

Experiments

Automated Systems Viewpoint



- Open System = plant to be controlled
  Uncontrollable events and Controllable events
- Goal: e.g. "level of the tank always between h and b"
  or "enforce the level of the tank above h"

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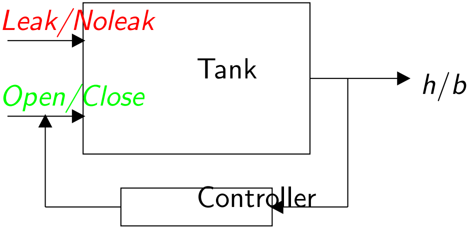

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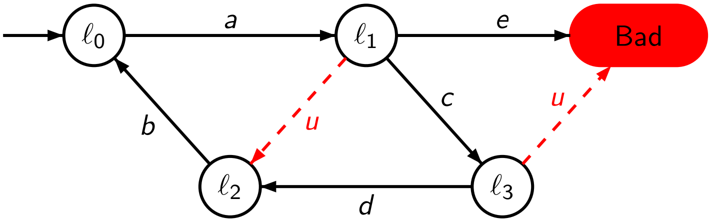
  or "enforce the level of the tank above h"
- Closed (loop) = Controlled System

Control Problems as Games



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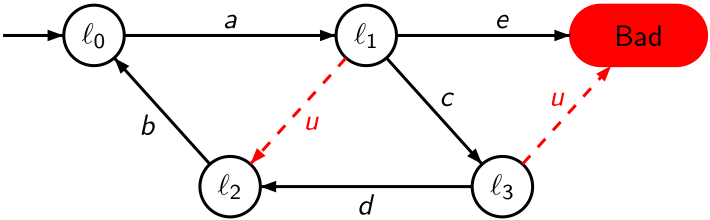

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  - Finite or pushdown or counter automata ...
  - Timed (or hybrid) automata

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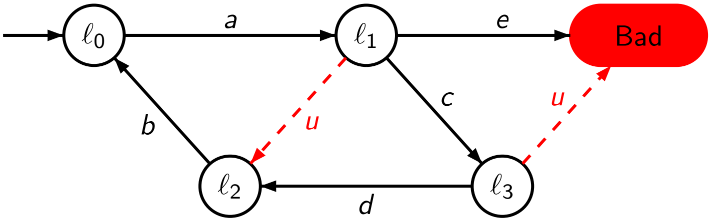

- Control problem = game = controller (C) vs environment (E)
- Various types of game models for C and E
  - Finite or pushdown or counter automata . . .
  - Timed (or hybrid) automata
- Goal: find a strategy for the controller to win
  Avoid bad states: safety control
  Enforce good states: reachability control

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Control Problems

Reachability Control

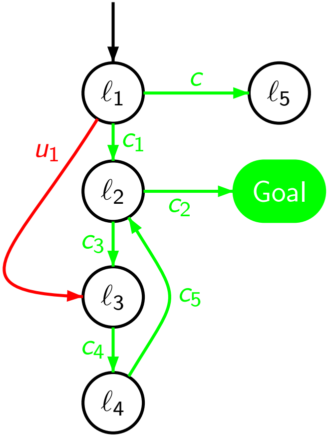
- Finite Games & Backward Algorithm
- Timed Games & Backward Algorithm
- Summary of the Results for Reachability Control

On-the-fly Algorithms for Reachability Control

Implementation, Optimizations, Time Optimality

Experiments

Backward Computation of Winning States



Controllable Predecessors:

$$\pi(X) = \big(\text{cPred}(X) \setminus \text{uPred}(\overline{X})\big)$$

Iterate π:

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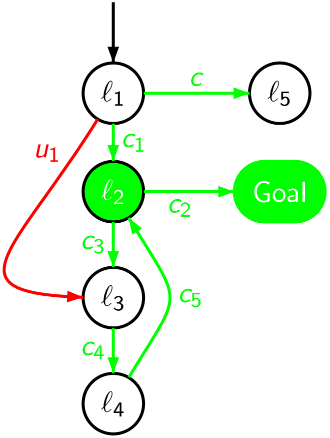

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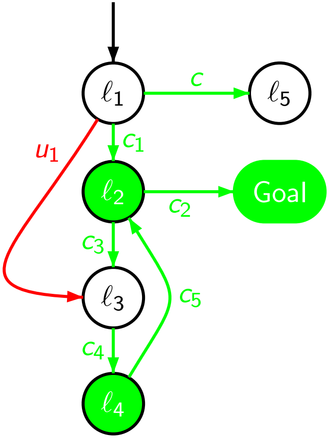

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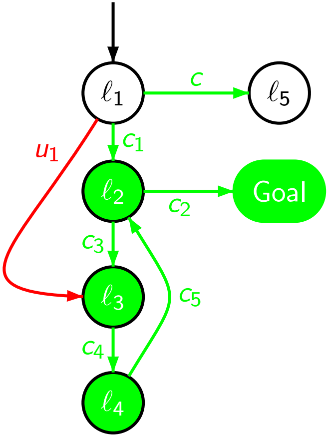

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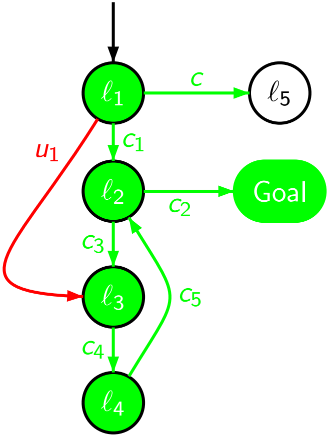

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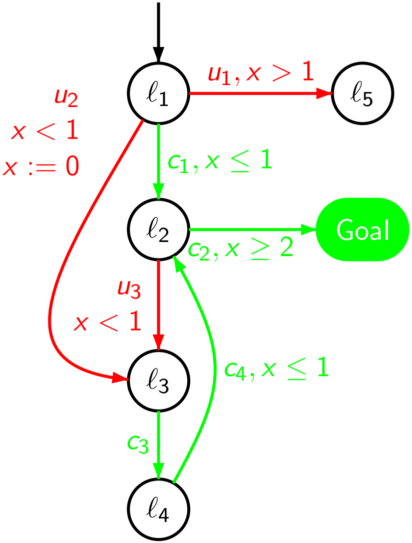
Reachability Control for Timed Games



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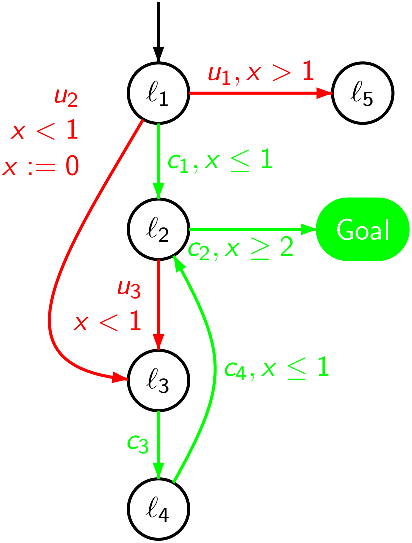

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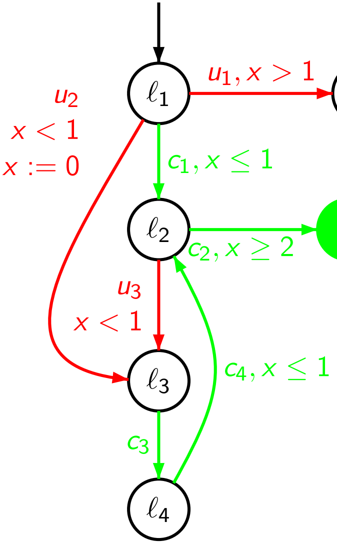

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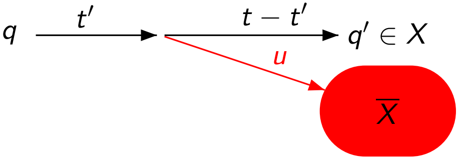

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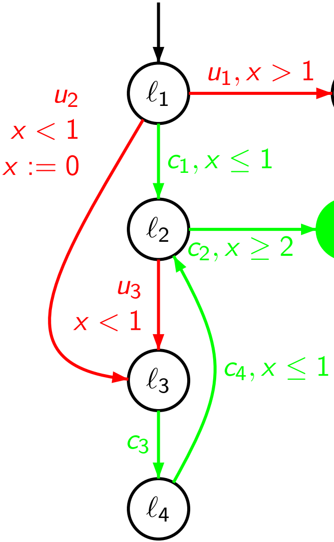

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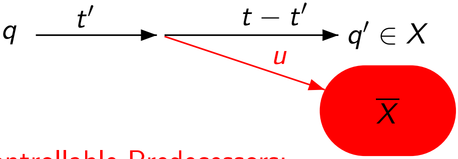

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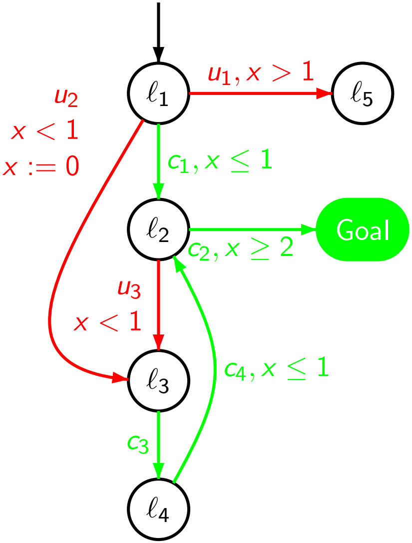

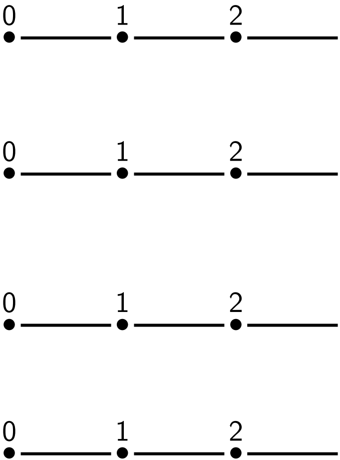

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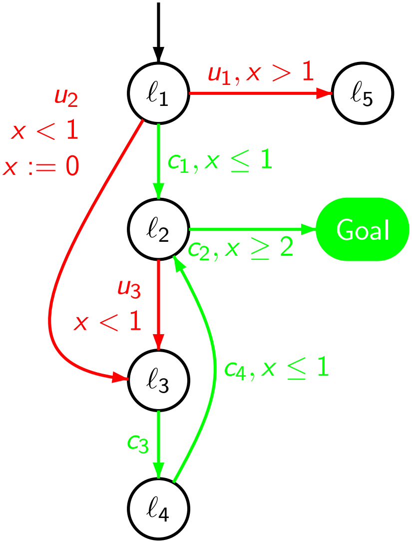

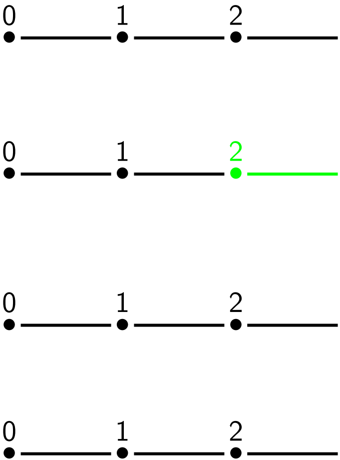

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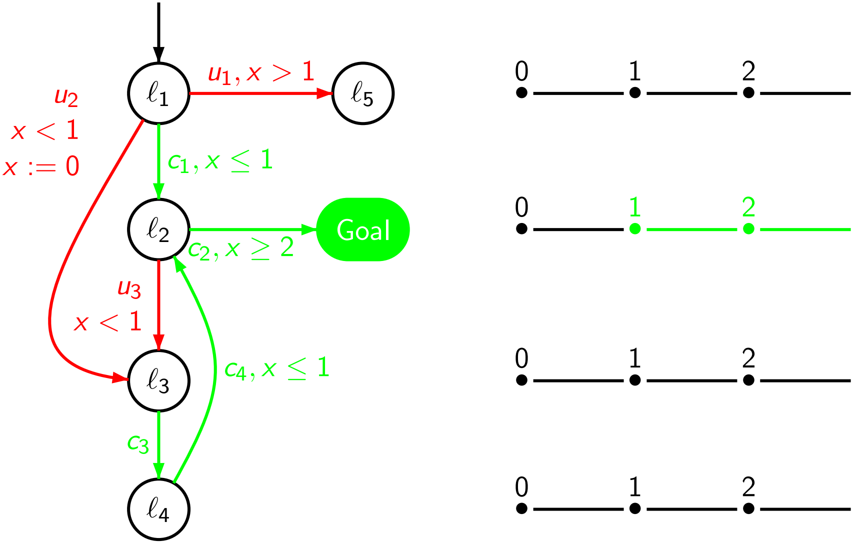

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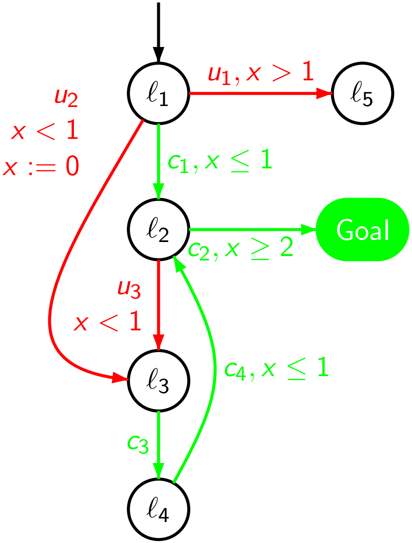

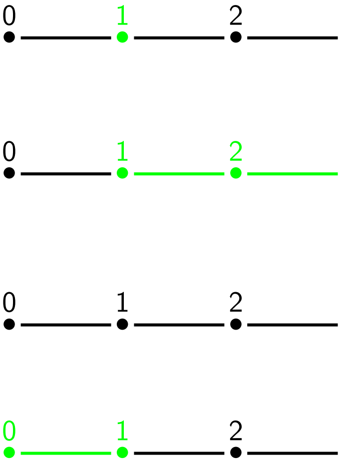

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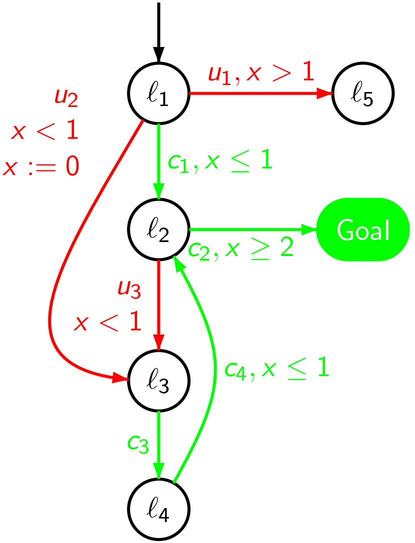

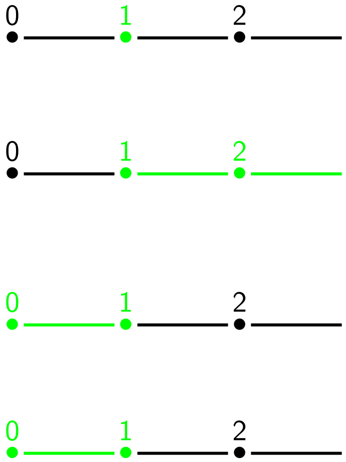

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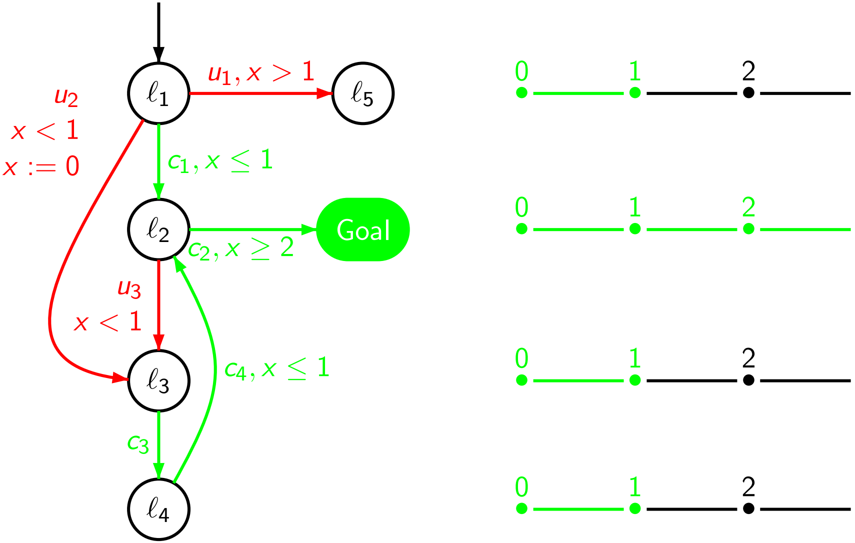

State-of-the-art

Known Results for Timed Games:

- Reachability in *Timed Automata* (Alur & Dill, 1994)

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- Advantages:
  - avoid constructing all backwards & forwards reachable states
  - allows for use of discrete variables (e.g. $i := i + 1$)
- Extends to Time (sub)-Optimal Control
- Efficient implementation with UPPAAL libraries

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On-the-fly Algorithms for Reachability Control
- Finite State Games
- Timed Games

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Experiments

Liu & Smolka Algorithm [Liu (ICALP'98)]

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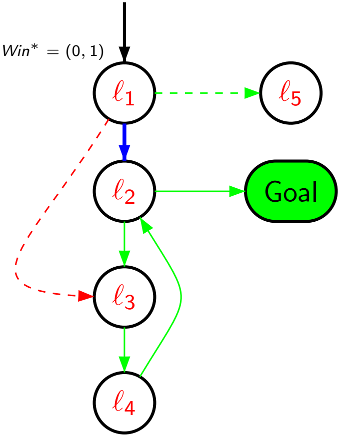

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$Depend[q_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge Win[q_0] \neq 1))$ **do**
  $e = (q, \alpha, q') \leftarrow pop(Waiting)$;
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    $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
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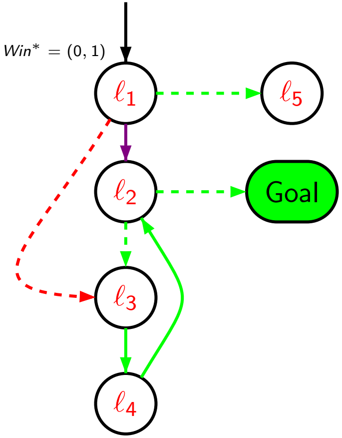

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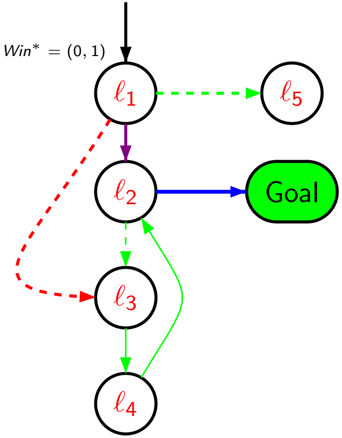

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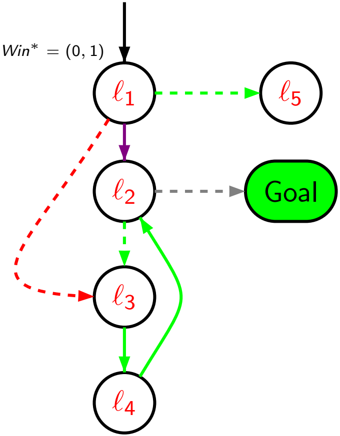

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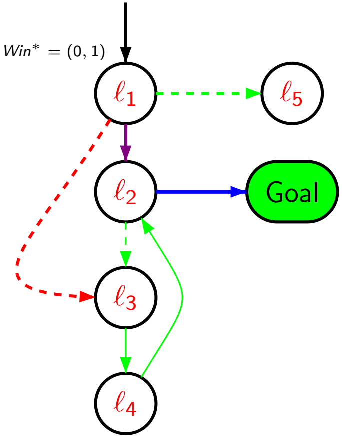

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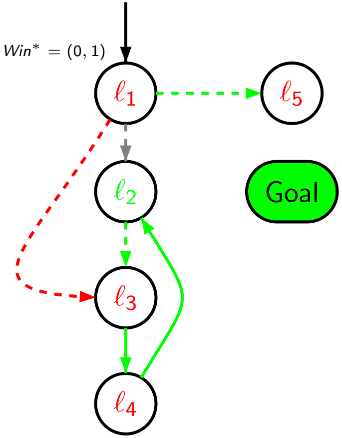
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in \text{Goal}\ ?\ 1 : 0)$;
$Depend[q_0] \leftarrow \emptyset$;

Main:

while $((Waiting \neq \emptyset) \wedge Win[q_0] \neq 1)$ **do**
  $e = (q, \alpha, q') \leftarrow pop(Waiting)$;
  if $q' \notin Passed$ **then** {
    $Passed \leftarrow Passed \cup \{q'\}$;
    $Depend[q'] \leftarrow \{(q, \alpha, q')\}$;
    $Win[q'] \leftarrow (q' \in \text{Goal}\ ?\ 1 : 0)$;
    $Waiting \leftarrow Waiting \cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if $Win[q']$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
  }
  else (* reevaluate *)
    $Win^*[q] \leftarrow$ **Update**$(Win^*[q])$;
    if $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then** {
      $Waiting \leftarrow Waiting \cup Depend[q]$;
      $Win[q] \leftarrow 1$;
      }
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
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    $Win[q'] \leftarrow (q' \in \text{Goal}\ ?\ 1 : 0)$;
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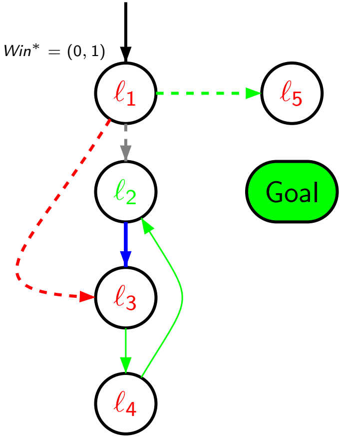
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
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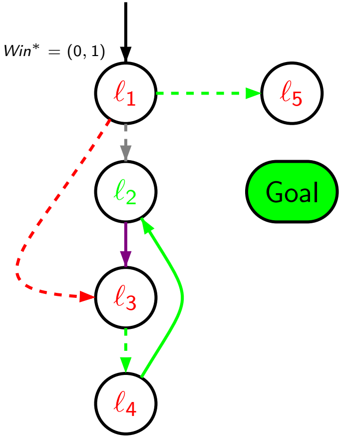
}
if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
endif
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
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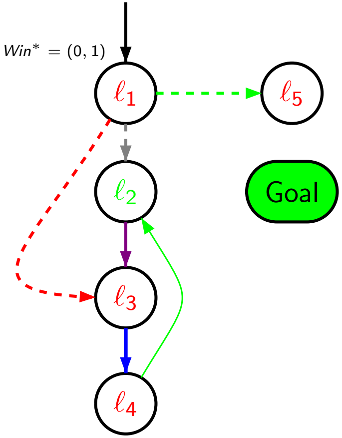
$Win[q] \leftarrow 1$;
}
if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
endif
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in \text{Goal} ~?~ 1 : 0)$;
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Main:

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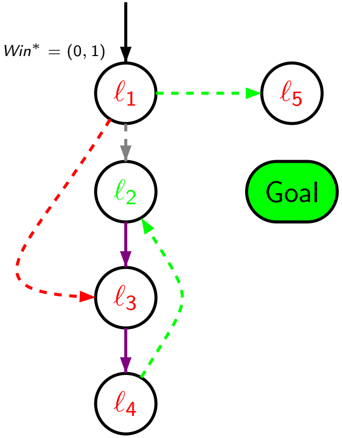
    $Win[q'] \leftarrow (q' \in \text{Goal} ~?~ 1 : 0)$;
    $Waiting \leftarrow Waiting \cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if $Win[q']$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
  }
  else (* reevaluate *)
    $Win^*[q] \leftarrow \textbf{Update}(Win^*[q])$;
    if $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then** {
      $Waiting \leftarrow Waiting \cup Depend[q]$;
      $Win[q] \leftarrow 1$;
      }
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



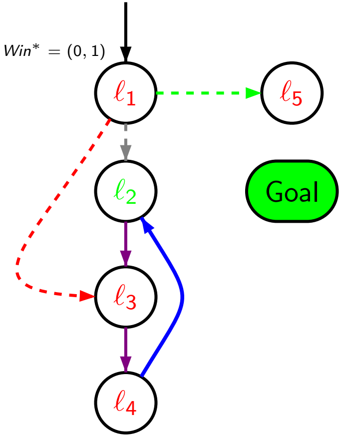
Initialization:

$Passed \leftarrow \{q_0\}$;
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in \mathrm{Goal}\ ?\ 1 : 0)$;
$Depend[q_0] \leftarrow \emptyset$;

Main:

```
while ((Waiting ≠ ∅) ∧ Win[q0] ≠ 1)) do
    e = (q, α, q') ← pop(Waiting);
    if q' ∉ Passed then {
        Passed ← Passed ∪ {q'};
        Depend[q'] ← {(q, α, q')};
        Win[q'] ← (q' ∈ Goal ? 1 : 0);
        Waiting ← Waiting ∪ {(q', α, q'') | q' --α--> q''};
        Win*[q] ← (0, #{q --u-->});
        if Win[q'] then Waiting ← Waiting ∪ {e};
    }
    else (* reevaluate *)
        Win*[q] ← Update(Win*[q]) ;
        if (Win*[q] = (k, 0) ∧ k ≥ 1) then {
            Waiting ← Waiting ∪ Depend[q];
            Win[q] ← 1;
            }
        if Win[q'] = 0 then Depend[q'] ← Depend[q'] ∪ {e};
    endif
endwhile
```

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
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}
if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
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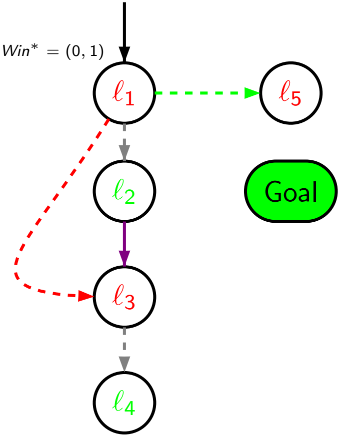
Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
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}
if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
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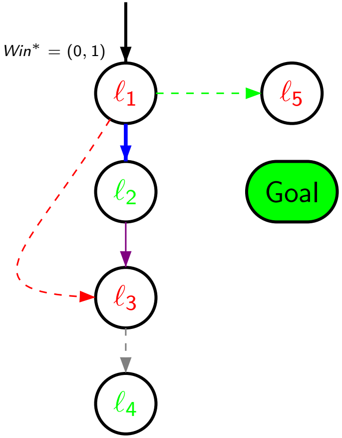
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in \text{Goal}\ ?\ 1 : 0)$;
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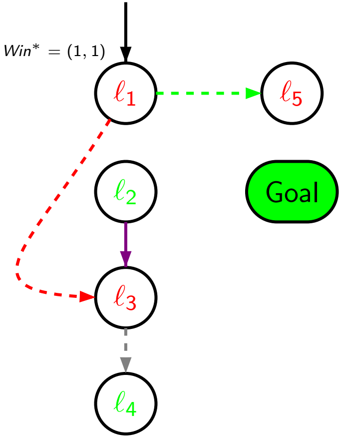
Main:

```
while ((Waiting ≠ ∅) ∧ Win[q0] ≠ 1)) do
   e = (q, α, q') ← pop(Waiting);
   if q' ∉ Passed then {
      Passed ← Passed ∪ {q'};
      Depend[q'] ← {(q, α, q')};
      Win[q'] ← (q' ∈ Goal ? 1 : 0);
      Waiting ← Waiting ∪ {(q', α, q'') | q' −α→ q''};
      Win*[q] ← (0, #{q −u→});
      if Win[q'] then Waiting ← Waiting ∪ {e};
   }
   else (* reevaluate *)
      Win*[q] ← Update(Win*[q]) ;
      if (Win*[q] = (k, 0) ∧ k ≥ 1) then {
         Waiting ← Waiting ∪ Depend[q];
         Win[q] ← 1;
         }
      if Win[q'] = 0 then Depend[q'] ← Depend[q'] ∪ {e};
   endif
endwhile
```

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in \text{Goal}\ ?\ 1 : 0)$;
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Main:

```
while ((Waiting ≠ ∅) ∧ Win[q0] ≠ 1)) do
    e = (q, α, q′) ← pop(Waiting);
    if q′ ∉ Passed then {
        Passed ← Passed ∪ {q′};
        Depend[q′] ← {(q, α, q′)};
        Win[q′] ← (q′ ∈ Goal ? 1 : 0);
        Waiting ← Waiting ∪ {(q′, α, q′′) | q′ --α--> q′′};
        Win*[q] ← (0, #{q --u-->});
        if Win[q′] then Waiting ← Waiting ∪ {e};
    }
    else (* reevaluate *)
        Win*[q] ← Update(Win*[q]) ;
        if (Win*[q] = (k, 0) ∧ k ≥ 1) then {
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        if Win[q′] = 0 then Depend[q′] ← Depend[q′] ∪ {e};
    endif
endwhile
```
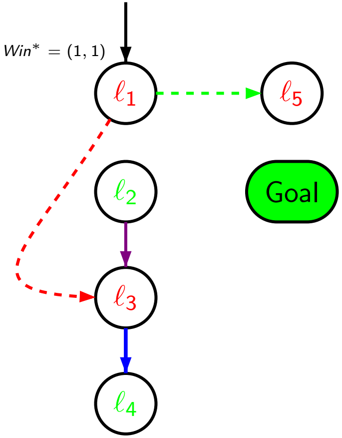

Liu & Smolka Algorithm [Liu (ICALP'98)]



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$Win[q_0] \leftarrow (q_0 \in \text{Goal}\ ?\ 1 : 0)$;
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Main:

while $((Waiting \neq \emptyset) \wedge Win[q_0] \neq 1))$ **do**
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    $Waiting \leftarrow Waiting \cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
    $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if $Win[q']$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
  }
  else (* reevaluate *)
    $Win^*[q] \leftarrow$ **Update**$(Win^*[q])$;
    if $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then** {
      $Waiting \leftarrow Waiting \cup Depend[q]$;
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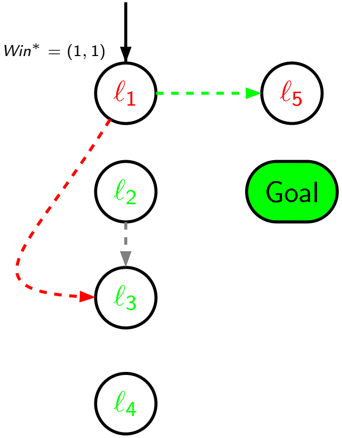
      }
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in \text{Goal}\ ?\ 1 : 0)$;
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Main:

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  if $q' \notin Passed$ **then** {
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    $Win[q'] \leftarrow (q' \in \text{Goal}\ ?\ 1 : 0)$;
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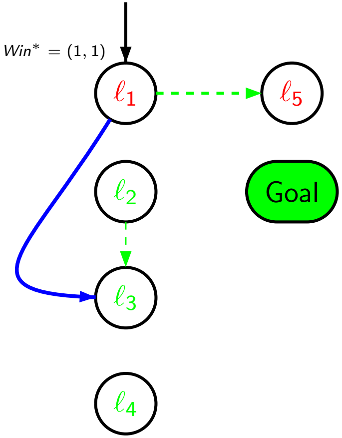
      $Win[q] \leftarrow 1$;
      }
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



Initialization:

$Passed \leftarrow \{q_0\}$;
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$Win[q_0] \leftarrow (q_0 \in \text{Goal}\ ?\ 1 : 0)$;
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    $Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
    if $Win[q']$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
  }
  else (* reevaluate *)
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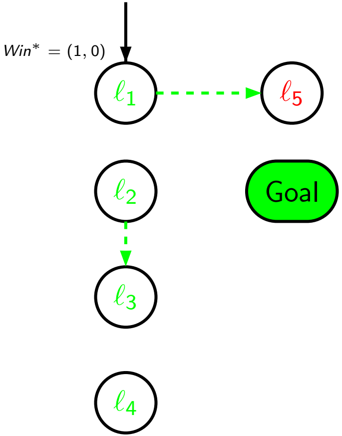
    if $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then** {
      $Waiting \leftarrow Waiting \cup Depend[q]$;
      $Win[q] \leftarrow 1$;
      }
    if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
  endif
endwhile

Liu & Smolka Algorithm [Liu (ICALP'98)]



Linear in # transitions

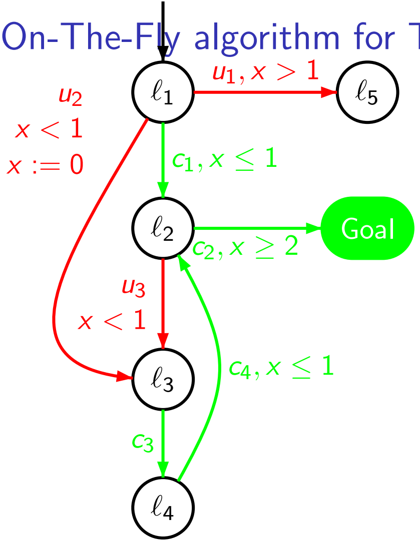
Initialization:

$Passed \leftarrow \{q_0\}$;
$Waiting \leftarrow \{(q_0, \alpha, q') \mid \alpha \in Act\ q \xrightarrow{\alpha} q'\}$;
$Win[q_0] \leftarrow (q_0 \in \text{Goal} ?\ 1 : 0)$;
$Depend[q_0] \leftarrow \emptyset$;

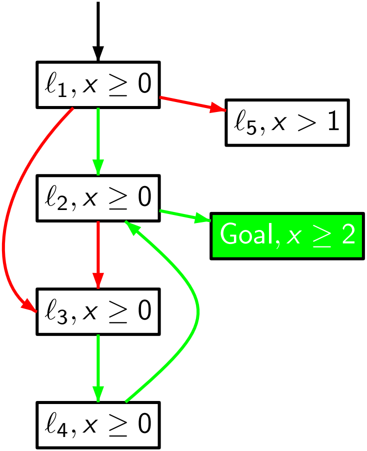
Main:

while $((Waiting \neq \emptyset) \wedge Win[q_0] \neq 1))$ **do**
$e = (q, \alpha, q') \leftarrow pop(Waiting)$;
if $q' \notin Passed$ **then** {
$Passed \leftarrow Passed \cup \{q'\}$;
$Depend[q'] \leftarrow \{(q, \alpha, q')\}$;
$Win[q'] \leftarrow (q' \in \text{Goal} ?\ 1 : 0)$;
$Waiting \leftarrow Waiting \cup \{(q', \alpha, q'') \mid q' \xrightarrow{\alpha} q''\}$;
$Win^*[q] \leftarrow (0, \#\{q \xrightarrow{u}\})$;
if $Win[q']$ **then** $Waiting \leftarrow Waiting \cup \{e\}$;
}
else (* reevaluate *)
$Win^*[q] \leftarrow$ **Update**$(Win^*[q])$;
if $(Win^*[q] = (k, 0) \wedge k \geq 1)$ **then** {
$Waiting \leftarrow Waiting \cup Depend[q]$;
$Win[q] \leftarrow 1$;
}
if $Win[q'] = 0$ **then** $Depend[q'] \leftarrow Depend[q'] \cup \{e\}$;
endif
endwhile

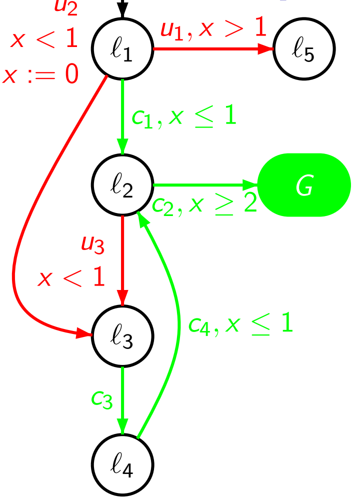
On-The-Fly algorithm for Timed Games: First Attempt



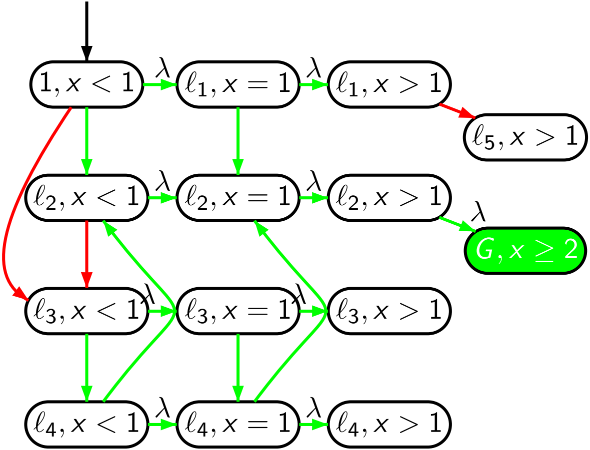
Using the Simulation Graph



Second Attempt [Tripakis (FM'99), Altisen (TOOLS'02)]



Stable Partitionning



Towards a True On-The-Fly Algorithm

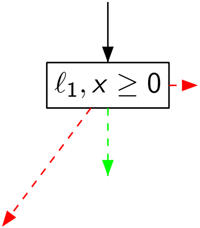
Our Approach:

- Write a Symbolic version of Liu & Smolka
- Use Symbolic states and Transitions
- Apply this to Timed Games

Key issues to be adressed:

- Symbolic States can be partially winning compared to FSG where 0 or 1
- When to propagate backward ?
- Termination, Complexity ?

Liu & Smolka for Timed Games



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$Win[S_0] \leftarrow S_0 \cap (\{\mathrm{Goal}\} \times \mathbb{R}^X_{\geq 0})$;
$Depend[S_0] \leftarrow \emptyset$;

Main:

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  $e = (S, \alpha, S') \leftarrow pop(Waiting)$;
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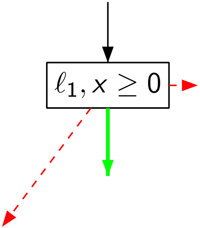
Skip algorithm

Liu & Smolka for Timed Games



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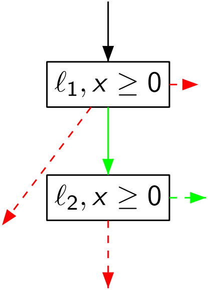

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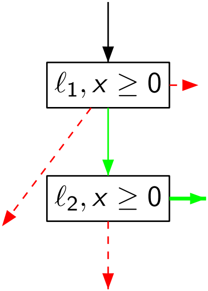

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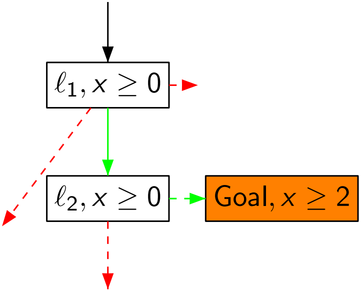

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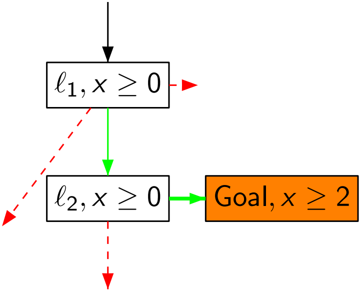

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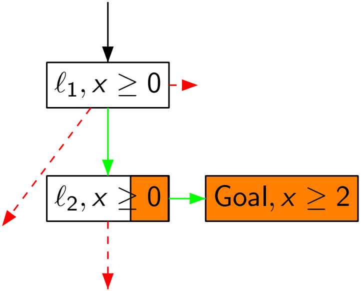

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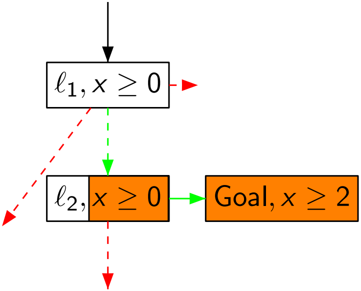

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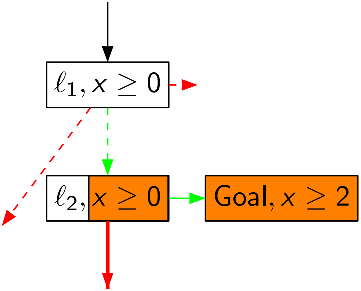

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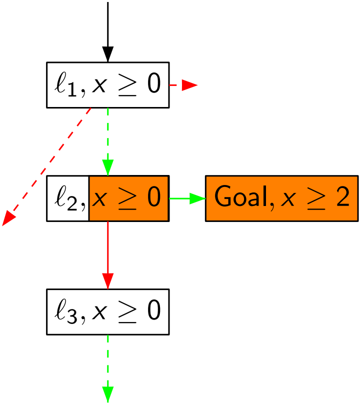

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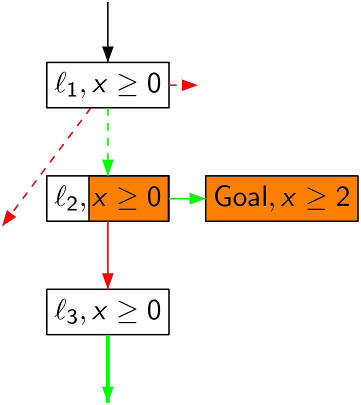

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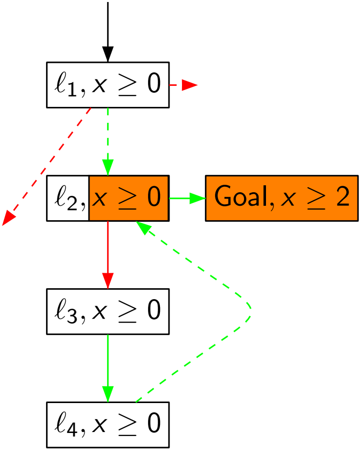

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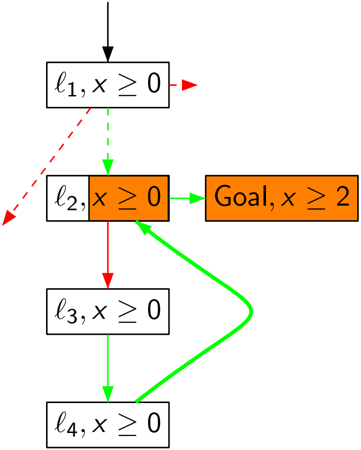

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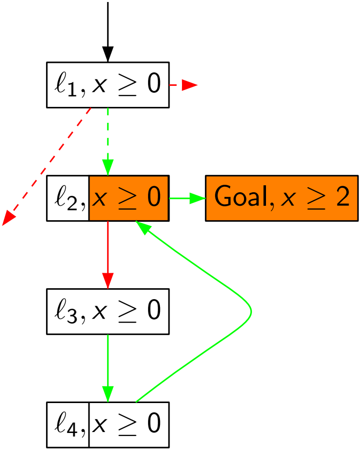
Skip algorithm

Liu & Smolka for Timed Games



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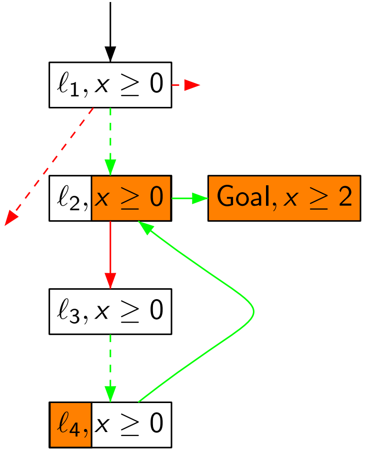

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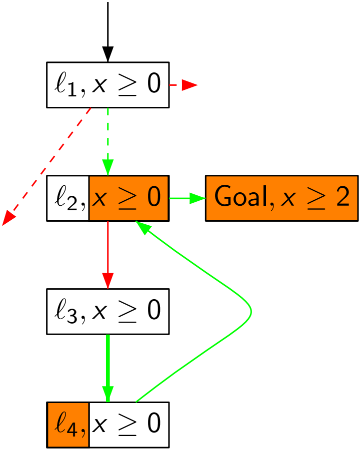

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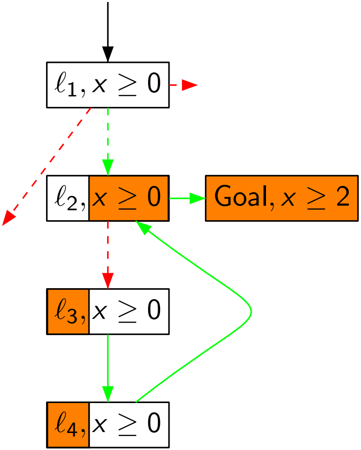

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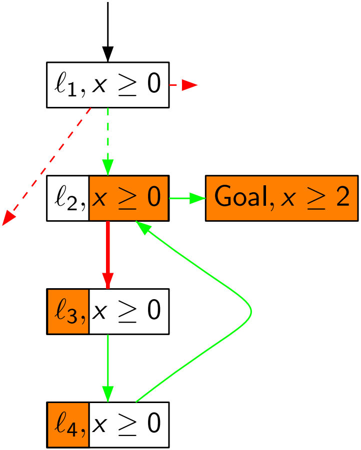

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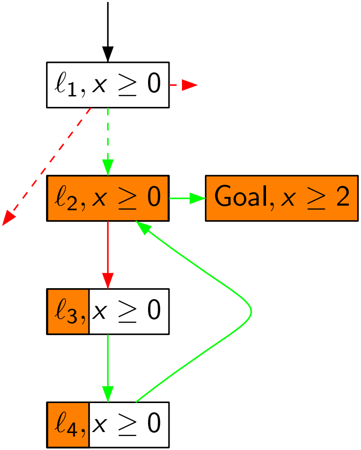

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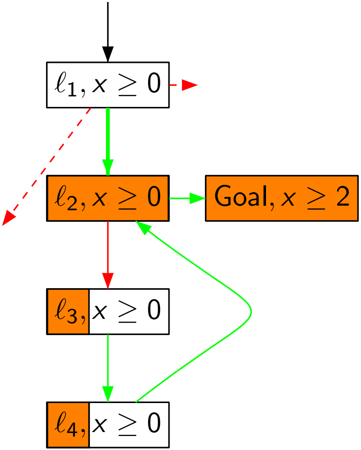

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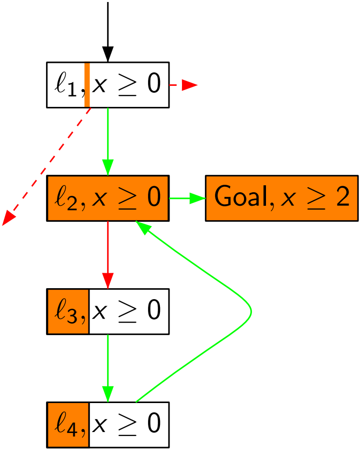

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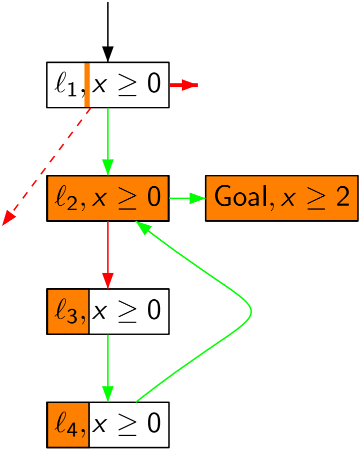

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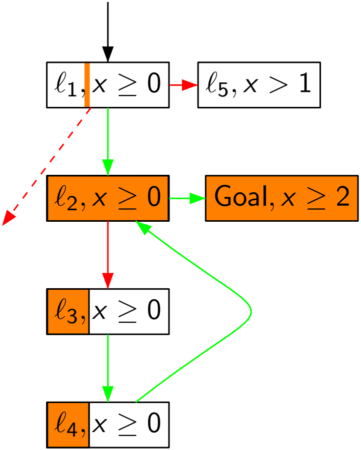

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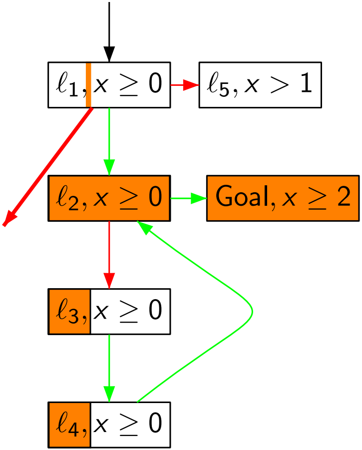

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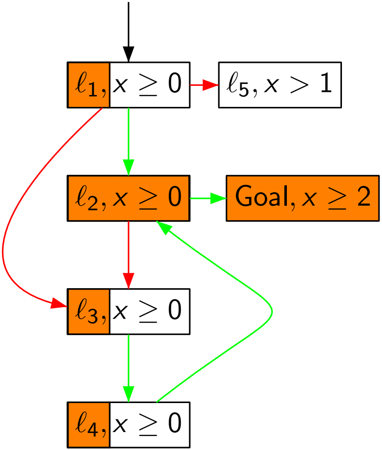

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Summary of the Results

- A True on-the-fly algorithm
  for reachability control and safety control
- Strategies can be computed
- Termination
  A symbolic edge (S, α, T)
  will be at most (1+ # regions(T)) times in *Waiting*
- Complexity
  A region may be in many symbolic states
  Our algorithm is Not linear in the size of the region graph
  Still it is theoretically optimal (EXPTIME)
- ... seems also good in practice!

Outline

Control Problems

Reachability Control

On-the-fly Algorithms for Reachability Control

Implementation, Optimizations, Time Optimality

Experiments

Efficient Implementation of Pred_t

Theorem

The following distribution law holds:

$$\mathrm{Pred}_t(\bigcup_i G_i, \bigcup_j B_j) = \bigcup_i \bigcap_j \mathrm{Pred}_t(G_i, B_j) \quad (1)$$

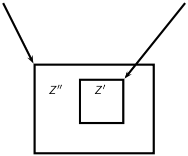
Theorem

If B is a convex set, then:

$$\mathrm{Pred}_t(G, B) = (G^{\swarrow} \setminus B^{\swarrow}) \cup ((G \cap B^{\swarrow}) \setminus B)^{\swarrow} \quad (2)$$

Inclusion Checking & Losing States

Inclusion checking



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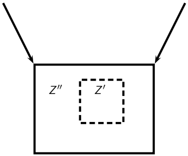

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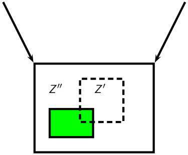
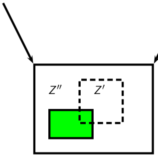
Inclusion checking

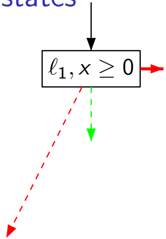


Inclusion Checking & Losing States

Inclusion checking

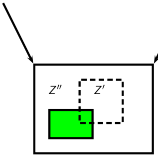


Losing states

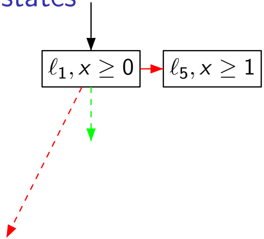


Inclusion Checking & Losing States

Inclusion checking



Losing states



Inclusion Checking & Losing States

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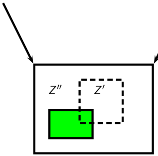
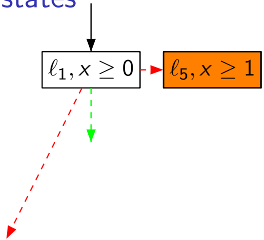
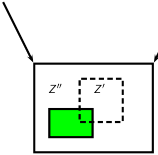


Losing states



Inclusion Checking & Losing States

Inclusion checking



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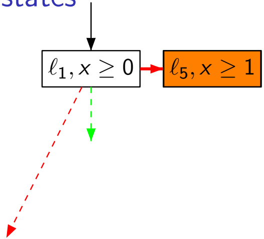
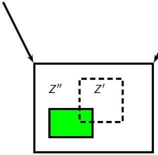
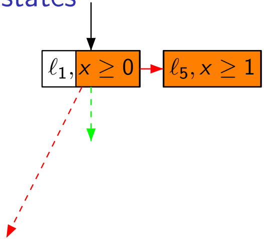


Inclusion Checking & Losing States

Inclusion checking

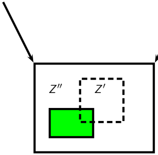


Losing states

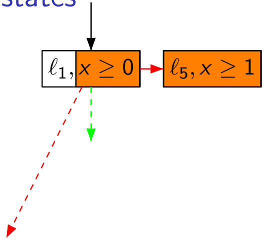


Inclusion Checking & Losing States

Inclusion checking



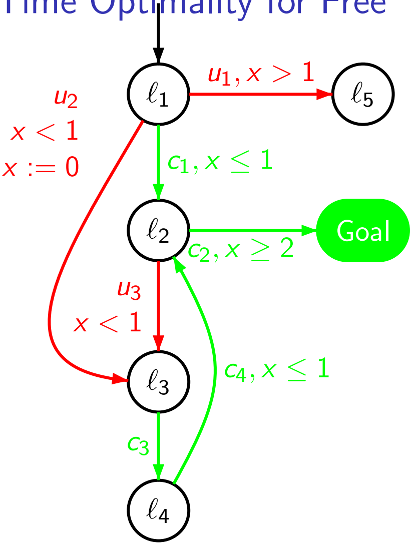
Losing states



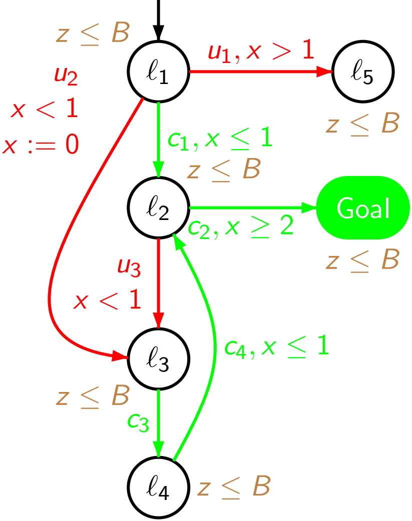
Pruning

Main:
while $((Waiting \neq \emptyset) \wedge (s_0 \notin Win[S_0]))$ **do**
  $e = (S, \alpha, S') \leftarrow pop(Waiting)$;
  if $Win[S] \subsetneq S$ **then**
    if $S' \notin Passed$ **then**
      $(\ldots)$
    else (* reevaluate *)
      $(\ldots)$
    endif
  endif
endwhile

Time Optimality for Free



Time Optimality for Free

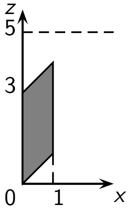


Assume:

- The initial state is winning
- We know an upper bound B of the optimal time needed to reach the winning state

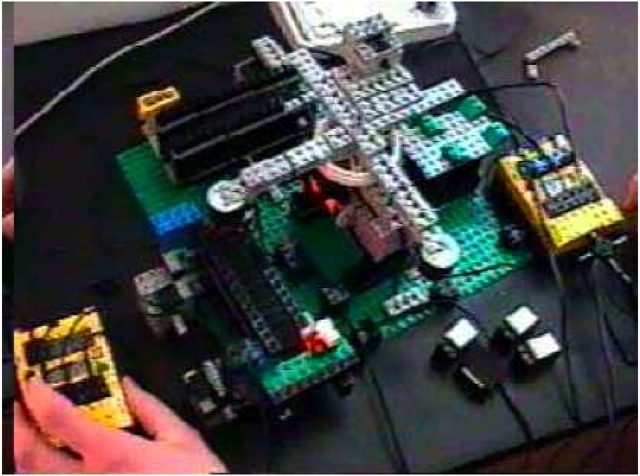
To compute the optimal time:

- Add a clock z (unconstrained at the beginning)
- Add a global invariant $z \leq B$

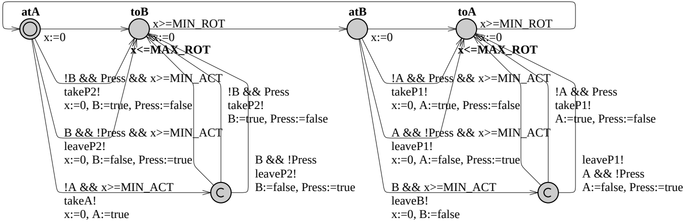


Outline

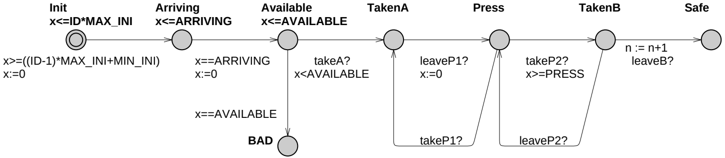
Experimental Results



Model of the robot



Model of the plates



Comparisons of Different Optimizations

Plates		Basic		Basic +inc		Basic +inc +pruning		Basic+lose +inc +pruning		Basic+lose +inc +topt	
		time	mem	time	mem	time	mem	time	mem	time	mem
2	win	0.0s	1M	0.0s	1M	0.0s	1M	0.0s	1M	0.04s	1M
	lose	0.0s	1M	0.0s	1M	0.0s	1M	0.0s	1M	n/a	n/a
3	win	0.5s	19M	0.0s	1M	0.0s	1M	0.1s	1M	0.27s	4M
	lose	1.1s	45M	0.1s	1M	0.0s	1M	0.2s	3M	n/a	n/a
4	win	33.9s	1395M	0.2s	8M	0.1s	6M	0.4s	5M	1.88s	13M
	lose	-	-	0.5s	11M	0.4s	10M	0.9s	9M	n/a	n/a
5	win	-	-	3.0s	31M	1.5s	22M	2.0s	16M	13.35s	59M
	lose	-	-	11.1s	61M	5.9s	46M	7.0s	41M	n/a	n/a
6	win	-	-	89.1s	179M	38.9s	121M	12.0s	63M	220.3s	369M
	lose	-	-	699s	480M	317s	346M	135.1s	273M	n/a	n/a
7	win	-	-	3256s	1183M	1181s	786M	124s	319M	6188s	2457M
	lose	-	-	-	-	16791s	2981M	4075s	2090M	n/a	n/a

Even better results are coming soon!

Conclusion and Future work

Conclusions:

- Successful development of a truly on-the-fly algorithm for reachability and safety games
- Efficient implementation using the UPPAAL DBM library
- Promising experimental results

Future work:

- Integration with the UPPAAL GUI.
- Guiding of the exploration by ordering the waiting list (heuristics) e.g.: Application to Job-Shop problems
- Distributed implementation
- Extension to deal with Partial observability criteria

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